**Math 300A Quiz 1**

**Question 1:** Prove by induction: For all integers n ≥1,

$$0 \times 1 + 1 \times 2 + 2 \times 3 + \ldots + (n-1)n = \frac{(n-1)n(n+1)}{3}$$

Or if you prefer this statement in sigma notation: $\sum_{k=1}^{n}(k-1)k = \frac{(n-1)n(n+1)}{3}$

**PROOF:** Proof by induction.

The statement above is true when n= 1:

For (n-1)n = 0*1 = 0 and also (1/3)(n-1)n(n+1) = (1/3)*(-1)*0*1 = 0.

Inductive step: Assume theorem is true for n and prove for n+1, so

**Assume True:** $0 \times 1 + 1 \times 2 + 2 \times 3 + \ldots + (n-1)n = \frac{(n-1)n(n+1)}{3}$

**Then Prove:** $0 \times 1 + 1 \times 2 + 2 \times 3 + \ldots + (n-1)n + n(n+1) = \frac{n(n+1)(n+2)}{3}$

The LHS of this equation is $(0 \times 1 + 1 \times 2 + 2 \times 3 + \ldots + (n-1)n) + n(n+1)$ which by the induction assumption equals

$$\frac{(n-1)n(n+1)}{3} + n(n+1) = n(n+1)\left(\frac{(n-1)+3}{3}\right) = \frac{n(n+1)(n+2)}{3}$$

and this equals the right hand side of the equation to be proved. QED

**Question 2:** Prove by induction:

For all integers n ≥1, the integer $5^n - 1$ is divisible by 4.

**PROOF:** Proof by induction.

The statement above is true when n= 1:

For $5^1 - 1 = 4$, which is divisible by 4.

**Assume True:** $5^n - 1$ is divisible by 4. This means that $5^n - 1 = 4a$ for an integer a.

**Then Prove:** $5^{n+1} - 1$ is divisible by 4. This means to prove that $5^{n+1} - 1 = 4b$, for some integer b.

The integer $5^{n+1} - 1 = 55^n - 1$ and by the induction hypothesis, this equals

5(1 + 4a) – 1 = 5 + 4*5a – 1 = 4 + 4*5a = 4(1+5a). Let the integer b = 1+5a. This shows $5^{n+1} - 1 = 4$ b. QED.