Math 300A Winter 2011

DEFINITION: Given two integers m and n, an integer k is a **common divisor of m and n** if k divides m and also k divides n. (In other words, there are integers a and b so that m = ka and n = kb.)

DEFINITION: The **greatest common divisor of m and n** is a positive integer d such that d is a common divisor of m and n and any other divisor k of m and n also divides m. **The greatest common divisor of m and n is denoted by gcd(m,n).**

DEFINITION: Two integers m and n are said to be **relatively prime** if their greatest common divisor is 1.

Examples: The gcd(150, 45) is 15, since 150 = 2*3*5*5 and 45 = 3*3*5. The gcd(49, 39) = 1 since 49 = 7*7 and 39 = 3*13.

Assignment 9 (due Wednesday, 3/9)

Problem 9-1: Post at least one of these to the Piazzza site.

- Submit the answer to a homework problem from a previous homework (no duplicates unless your answer is significantly different).
- Comment on another student's proof.
- Post a question whose answer you care about.
- Answer another students question.

Problem 9-2: Prove this theorem that we have been using.

For any integers m and n, there is an integer q and an integer r, with $0 \le r < |m|$ so that n = qm + r.

Hint: Induction on n.

Problem 9-3: Prove: If m, n, q, and r and integers, then the set of common divisors of m and n is the same as the set of common divisors of m and r.

What does this mean if r = 0. Is the theorem still true?

Conclude as a corollary: gcd(m,n) = gcd (m,r).

Problem 9-4: Use the result of 9-3 for an algorithm to find the gcd of any two integers that does not require factoring them into prime factors.

Then use your algorithm on some non-obvious numbers, including some of at least 4 digits and preferably more.

Hint 1: The first step is n = qm + r, with gcd(m,n) = gcd (m,r). The next step is m = q'r + r'. With gcd (m,r) = gcd (r, r'). Notice that the remainder is smaller at each step, so at some point it must be zero. What does this tell you when you get to that point?
Hint 2: Try out these steps (and continue them) with some simple examples, such as n = 150 and m = 45.

Problem 9-5:

Any function X from N to R defines a sequence of real numbers, $X_1, X_2, \ldots$

The number A is defined to be the limit of X_n as $n \to \infty$ if this is true:

For every $\varepsilon > 0$, there is a positive integer N so that for all $n > N$, $|X_n - A| < \varepsilon$.

Notation: $\lim_{n\to\infty} X_n = A$

PROVE: If $X_n = \dfrac{2n-1}{n}$, then $\lim_{n\to\infty} X_n = 2$.

Problem 9-6:

Suppose that Z_n is a sequence for which this is true: There is a positive integer N such that for every $\varepsilon > 0$, for all $n > N$, $|Z_n - A| < \varepsilon$.

What would an example of such a Z_n be? What can you prove about Z_n that must be true.

Problem 9-7:

(a) Write what it means for a number A *not* to be the limit of X_n as $n \to \infty$. In other words, what is the negation of the definition in 9-5.

(b) If the sequence $Y_n = (-1)^n$ for all positive integers n, prove that there is no number A that is the limit of Y_n as $n \to \infty$.