Math 300 Assignment 8 (due Wednesday, 3/2)

Quiz 3 on Friday, 3/4

Next Extra Credit will be announced in email and then will be online.

Notation: Z denotes the set of integers, R denotes the set of real numbers, Z_n denotes the integers mod n.

Problem 8-1

a) Let $f = \{(x,y) \in Z \times Z : 3x + y = 4\}$. Is this a function from Z to Z? Explain.
b) Let $g = \{(x,y) \in Z \times Z : x + 3y = 4\}$. Is this a function from Z to Z? Explain.

Problem 8-2

a) Let $u = \{(x^2, x) : x \in R\}$. Is this a function from R to R? Explain.
b) Let $v = \{(x^3, x) : x \in R\}$. Is this a function from R to R? Explain.

Problems 8-3 and 8-4 concern permutations of a set S. Recall that a permutation of S is a 1-1 and onto function of S to S. Recall that for two permutations, the composition is written as a product. For example, $\mu\nu = \mu \circ \nu$. So in particular, $\mu^2 = \mu \circ \mu$.

Problem 8-3

Suppose μ is a permutation of S = {1, 2, 3, 4, 5}. This permutation can be defined as a set of 5 ordered pairs, but we introduced a shorter notation – an ordered list of elements of S: [μ(1) μ(2) μ(3) μ(4) μ(5)].

For example, μ = [2 3 1 5 4] means that μ(1) = 2, μ(2) = 3, μ(3) = 1, μ(4) = 5, μ(5) = 4.

a) If μ = [2 3 1 5 4], what is the symbol (i) for μ^2? (ii) For μ^3? (iii) For $(\mu^3)^2$?

b) Let μ be as above and ν = [5 2 3 4 1]. (i) What is the symbol for μν? (ii) What is the symbol for νμ?

c) Define the identity ι: S → S by ι(k) = k for all k in S. Is ι a permutation? If so, what is its symbol?

d) If μ is as above, what is the symbol for μ^{-1}?

Problem 8-4

The set Z_5, the integers mod 5, can be represented by symbols {0, 1, 2, 3, 4}.

For each element k of Z_5, define the multiplication map $m_k : Z_5 \to Z_5$, by the formula $m_k(x) = kx \pmod 5$.

a) For which of the $k \in \{0, 1, 2, 3, 4\}$ is m_k a permutation of Z_5? Show this.

b) For which of the $k \in \{0, 1, 2, 3, 4\}$ is $m_k^{-1} = m_k$? Show this.

The set Z_6, the integers mod 6, can be represented by symbols {0, 1, 2, 3, 4, 5}. For each element k of Z_6, define the multiplication map $m_k : Z_6 \to Z_6$, by the formula $m_k(x) = kx \pmod 6$.

c) For which of the $k \in \{0, 1, 2, 3, 4, 5\}$ is m_k a permutation of Z_6? Show this.

Problem 8-5: Gemignani, 7.4 #3, parts b, c, d, e

Problem 8-6: Gemignani, 7.5 #1

Problem 8-7: Gemignani, 7.5 #3

Problem 8-8: Let the function $f: R \to R$ be defined by $f(x) = (1+x)^2$. Let T be the interval [-1, 1]. (Recall that the interval [a, b] is the set $\{x: a \le x \le b\}$.)

For each question below, the answer is a set that should be specified as an interval or a union of intervals such as [*a*, *b*] for specific numbers *a* and *b*. (In other words, the answer should not have squares or square roots in it; it should be simplified to a collection of inequalities for x,)

a) What set is the set $f^{-1}(T)$? Justify your answer.

b) What set is the set $f(T)$? Justify your answer.

c) What set is the set $f(f^{-1}(T))$? Justify your answer.

d) What set is the set $f^{-1}(f(T))$? Justify your answer.

Hint: You can use algebra or you can use the graph of f for your justifications.