Math 300A: Winter 2011

Quiz 2 will be Friday, 2/25. The topic will be ***Functions*** (see reading assignment)

Reading Assignment in Gemignani

- Sections 7.3, 7.4 by Wednesday, 2/23 (needed for homework)
- Section 7.5 by Friday 2/25 (a bit may be on the quiz)

Assignment 7 (Due Wednesday 2/23)

Problem 7-1 (Fibonacci numbers)

The infinite sequence of integers $F_1, F_2, F_3, F_4, F_5, \dots F_n, \dots$ called the Fibonacci numbers is defined recursively in this way:

> Define $F_0 = 0, F_1 = 1$
>
> For any n > 1, define $F_n = F_{n-1} + F_{n-2}$
>
> Note: Some people skip F_0 and start by setting both F_1 and F_2 to be 1. This does not change any values for the numbers.

(a) Compute and write down the first 10 Fibonacci numbers.

(b) Prove that this equation is true for integers $n \geq 0$: $\sum_{k=0}^{n} F_k = F_{n+2} - 1$.

Problem 7-2:

- Answer Gemignani, Section 7.3 #1 and
- Also find the images of the 8 functions f_k in the same Example 9.

Problem 7-3:

- Answer Gemignani, Section 7.3 #2 for only (b), (c) and (d).

Also, answer the same questions for these rules:

- k(n) = n^2, where S = Z_3 the integers modulo 3, a set with 3 elements.
- O(c) = center of c, where S is the set of circles in the plane.
- E(L) = (m, b), where S is the set of lines in the (x, y) plane and y = mx + b is the equation of the line.

Problem 7-4: Gemignani, Section 7.3 #3 (in your own words, please)

Problem 7-5: Gemignani, Section 7.3 #4

Problem 7-6: Find and list all of the one-one and onto functions from the set S = (a, b, c) into itself. (Look at Gemignani Section 7.4 #1 for a hint.)

Problem 7-7: Gemignani, Section 7.4 #2, plus one more: (h) f(n) = n if n is an even integer and f(n) = -n if n is an odd integer.

Problem 7-8: Gemignani, Section 7.4 #4

Extra Credit (each with different due dates – do not attach to your regular homework!)

Extra 7-A (10 points, **due Wednesday 2/23)**

Write out by hand completely and correctly (and legibly) all four definitions found in Sections 7.3 and 7.4 of Gemignani.

Extra 7-B (20 points, **due Friday 2/25)** (Related to the Fibonacci Numbers)

(a) Let $\phi = \frac{1+\sqrt{5}}{2}$. Verify that ϕ and $1-\phi$ are the two solutions of the quadratic equation $x^2 = x+1$, which can be written in standard form as $x^2 - x - 1 = 0$.

(b) Let $S_n = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$. Use a calculator or spreadsheet or other software to compute accurately the first 10 of the numbers S_n in this sequence. Write them down. How are these numbers related to the Fibonacci sequence?

(c) Use the sequence S_n to prove a formula for the n^{th} Fibonacci number.

(d) Use this formula to calculate the 35^{th}, 75^{th} and 105^{th} Fibonacci numbers. Also, compute $\frac{\phi^n}{\sqrt{5}}$ for n = 35, 75, and 105. What do you observe? How do you explain this?

Extra 7-C (20 points, **due Monday 2/28)** (Modular arithmetic)

Important standard notation: Z_n denotes the arithmetic of the integers modulo n. Specifically the set Z_n is the set of equivalence classes of the integers modulo n, which are the "numbers" in this arithmetic.

In problem 6-6 you constructed multiplication tables for the integers mod 4 and mod 5, in other words for Z_4 and Z_5 in this new notation. Let f be the function from Z_5 to itself defined by the rule f(a) = 2a. Let g be the function from Z_4 to itself defined by the rule g(a) = 2a. Let h be the function from Z_5 to itself defined by the rule h(a) = 3a. Let k be the function from Z_4 to itself defined by the rule k(a) = 3a. Tell which of these functions are one-one and onto and which are not. Explain the reasons clearly in each case.