Math 300A: Winter 2011

Assignment 6 (Due Wednesday 2/16)

Problem 6-1 (Proof of formula for numbers in Pascal's triangle)

IMPORTANT NOTE about Notation: When we developed this in class, we used the notation that $(x+y)^n = \sum_{r+s=n} a_{r,s} x^r y^s$. So the subscripts on the numbers are the corresponding powers of x and y. This was done to make the recursive step clearer. However the Standard Notation for these numbers is either $C_{n,r}$ or $\binom{n}{r}$, and is pronounced verbally as "n choose r". So the relationship between the two is $C_{n,r} = a_{r,n-r}$. So we will switch to the standard notation.

Definition: For all nonnegative integers, n and r, define $C_{n,r}$ by (1) $C_{n,r} = 1$ if r = 0 or n and (2) $C_{n,r} = C_{n-1.r-1} + C_{n-1,r}$ for all n > 0.

Problem. Using only the definition, use induction to prove the formula $C_{n,r} = \frac{n!}{r!(n-r)!}$.

Problem 6-2

Use induction to prove that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.

Problem 6-3

Find a simple formula for the sum of the numbers in the n^{th} row of Pascal's triangle and prove it. (Note: The 1 at the top is considered the 0^{th} row).

Problem 6-4

Read the definition of equivalence relation in Gemignani, Section 8.2. (This concept will be discussed in class on Friday and Monday.)

Prove that the relation defined by Definition 8.4 is in fact an equivalence relation for pairs of natural numbers.

Problem 6-5

For any positive integer, we define a relation on all the integers called "congruence mod n". Two integers *a* and *b* are "congruent mod *n*" if the both have the same remainder when divided by n (or stating it another way, if *n* divides (*a*-*b*)).

-a- Prove that for any given *n*, this relationship is an equivalence relation.

-b- For a given *n*, prove that every integer is congruent mod *n* to one of the following numbers: 0, 1, …, *n*-1.

Problem 6-6

FACT: We will prove in class that if *a* is congruent to *a*' and *b* is congruent to *b*' mod n, then *a*+*b* is congruent to *a*'+*b*' mod *n* and also ab is congruent to *a*'*b*' mod *n*. This means that we can add or multiply congruence equivalence classes by adding or multiplying representatives. For instance, arithmetic mod 2 consists of addition and multiplication of the classes of 0 and 1. Arithmetic mod 3 is arithmetic on the symbols 0, 1,2, etc.

-c- Construct addition and multiplication tables for arithmetic mod 5 and arithmetic mod 4.