

Math 300 Assignment 5 (Due Wednesday, 2/9)

Note about first two problems: a similar proposition was assigned before to four students, but no one submitted a correct proof.

Problem 1. Prove this proposition.

Let p and q be distinct prime numbers. If n is an integer that is divisible by both p and q , then n is divisible by pq .

Problem 2. Prove this set equality. (Note: The symbol \mathbb{Z} denotes the set of all integers.)

$$\{n \in \mathbb{Z}: 3|n\} \cap \{n \in \mathbb{Z}: 5|n\} = \{n \in \mathbb{Z}: 15|n\}$$

Problem 3. For any set S , the power set of S – denoted $P(S)$ – is the set of all subsets of S .

Prove by mathematical induction: For any set, if $\#S = n$, then $\#P(S) = 2^n$.

Here is a more formal version of the same statement: For any finite set S with cardinal number n , then the cardinal number of $P(S) = 2^n$.

Problem 4. Prove by mathematical induction (if you happen to know a formula for the left sign of the inequality, do not assume the formula. Just use basic properties of inequalities and arithmetic and induction.

For all positive real numbers x , $(1+x)^n \geq 1 + nx$.

Problem 6. Problem 5 of Section 5.1 of Gemignano asks for a set diagram for 4 sets that can be used to show set relations. But the answer in the back of the book is an incorrect diagram that does not show all possibilities.

- (a) Draw the 3 different set diagrams that you already know, the ones for one, two or three sets. Count the number of regions inside each of these diagrams. (Hint: the number for 2 sets is 3). Then tell how many regions should be inside a correct diagram for 4 sets (with an explanation).
- (b) What is the largest number of regions in a diagram for 4 sets, where a circle represents each set? Show an example.
- (c) Either create your own correct diagram for 4 sets or go online to find one or two. If you do find an answer online, show the diagram and also cite the web address of your source.

Extra Credit Problems: These can be submitted later; some of the more historical questions should be answered with a word-processed document submitted as a computer file.

Extra Credit A: Tell how many regions should be in a set diagram for n sets. Give a clear explanation of your reasoning. Prove your claim by induction.

Extra Credit B: Write a short report on one or more other approaches to set diagrams. For example, a different approach to set diagrams based on squares was invented by Lewis Carroll (real name Charles Lutwidge Dodgson). Other approaches are based on other shapes. (A “short report” means more than a half-page and less than a page of word-processed document – not counting figures. And give citations for your sources).