Assignment 3 Answers

Problem 3.1 – Section 4.2 #4

a) False. $\{1, 2, 3\} \subset \{1, 2, 3, \{a, b\}\}$
b) True.
c) True.
d) True.

Problem 3.2 – Section 4.3 #2

The eight subsets of {1, 2, h} are
{}, {1}, {2}, {h}, {1, 2}, {2, h}, {1, h}, (1, 2, h}

Problem 3.3 – Section 4.4 #3

a) $x \in S \cap T \Leftrightarrow x \in S$ and $x \in T \Leftrightarrow x \in T$ and $x \in S \Leftrightarrow x \in T \cap S$.
b) $x \in S \Rightarrow x \in S$ or $x \in T \Rightarrow x \in S \cup T$, so $S \subset S \cup T$.
c) $x \in S \cap T \Leftrightarrow x \in S$ and $x \in T \Rightarrow x \in T \Leftrightarrow x \in T$, so $S \cap T \subset T$.
d) By c, $S \cap \{\} \subset \{\}$. But $\{\} \subset$ any set, since there are not elements in {}.
e) So to show $S \cup \{\} = S$, first note by (b) that $S \cup \{\} \supset S$. The statement $x \in \{\}$ is always false. So $x \in S \cup \{\} \Leftrightarrow x \in S$ or $x \in \{\} \Rightarrow x \in S$ is true since $x \in \{\}$ is false. Thus $S \cup \{\} \subset S$.
f) $x \in S \cap (T \cap W) \Leftrightarrow x \in S \wedge (x \in T \wedge x \in W) \Leftrightarrow (x \in S \wedge x \in T)\wedge x \in W$
$\Leftrightarrow x \in (S \cap T) \cap W$.
g) $x \in S \Leftrightarrow x \in S \vee x \in S \Leftrightarrow x \in S \cup S$ since $A \vee A = A$ is a tautology. So $S \cup S = S$.
h) $x \in S \Leftrightarrow x \in S \wedge x \in S \Leftrightarrow x \in S \cap S$ since $A \wedge A = A$ is a tautology. So $S \cap S = S$.
i) $x \in S \cup (T \cap W) \Leftrightarrow x \in S \vee (x \in T \wedge x \in W) \Leftrightarrow$
$(x \in S \vee x \in T) \wedge (x \in S \vee x \in W) \Leftrightarrow x \in (S \cup T) \cap (S \cup W)$.

Problem 3.4 – Section 4.5 #1
S = {5, 6, 7, 8, 9}. T = {4, 5, 6, 11, A}, W = {1, 5, A, B}

a) $S - T = \{7, 8, 9\} \subset \{5, 6, 7, 8 9\} = S$
b) $(S - T) \cap (T - S) = \{7, 8, 9\} \cap \{4, 11, A\} = \{\}$
c) $T - (S \cup W) = \{4, 5, 6, 11 A\} - (1, 5, 6, 7, 8, 9, A, B\} = \{4, 11\}$ and
$(T - S) \cap (T - W) = \{3, 11, A\} \cap \{4, 6, 11\} = \{4, 11\}$. So the sets are equal.
d) $T - (S \cap W) = \{4, 5, 6, 11 A\} - \{5\} = \{4, 6, 11 A\}$ and
$(T - S) \cup (T - W) = \{4, 11, A\} \cup \{4, 6, 11\} = \{4, 6, 11 A\}$. So the sets are equal.
e) $(T - S) - W = \{4, 11, A\} - \{1, 5, A, B\} = \{4, 11\}$ and
$T - (S - W) =\{4, 5, 6, 11, A\} - \{6, 7, 8, 9\} = \{4, 5, 11, A\}$ so the sets are unequal.
f) $(S \cap W) \cup (S - W) = \{5\} \cup \{6, 7, 8, 9\} = \{5, 6, 7, 8, 9\} = S$.

Problem 3.5 – Section 4.5 #6

S and T are sets. Is there a set X that solves each of these equations. (In other words, is the equation always solvable for any sets S and T, or is there a counterexample.)

a) $S \cup X = T$. Counterexample: If T is a proper subset of S (a subset of $S \cup X$), then T is also a proper subset of $S \cup X$ and thus the sets are not equal.
b) $S \cap X = T$. Counterexample: If S is a proper subset of T, then $S \cap X$ is a subset of S and so is also a proper subset of T and thus the sets are not equal.
c) $S - X = T$. Counterexample: If S is a proper subset of T, then $S - X$ is a subset of S and so is also a proper subset of T and thus the sets are not equal.
d) $X - S = T$. Counterexample: Any element of $S \cap T$ is an element of T but not an element of $X - S$. So for any example where $S \cap T$ is not empty, then there is no X.

There are conditions on S and T that are necessary and sufficient for there to be a solution X, but these are not included in this answer sheet.