

Answers to Some of Assignment 2

Part B. Contrapositive

Note: There are a number of correct ways to phrase each of these contrapositive statements, so your answer can be different and may still be correct. Since the original statements are all true theorems, then so should your statement be true.

For each statement below, write the contrapositive.

- a) All equilateral triangles are isosceles

If a triangle is not isosceles, then it is not equilateral.

(You can rephrase the original as: If a triangle is equilateral, then the triangle is isosceles.)

- b) Any two equilateral triangles are similar.

If two triangles are not similar, then one of them is not equilateral.

Or

If a pair of triangles is not similar then this is not a pair of equilateral triangles.

- c) For real numbers a, b, c , if $a = b$ then $ac = bc$.

For real numbers a, b, c , if ac is not equal to bc , then a is not equal to b .

- d) In a right triangle, two angles are complementary.

In a triangle, if no two angles are complementary then the triangle is not a right triangle.

Or

If a triangle has no pair of complementary angles, then the triangle is not a right triangle.

- e) For integers m and n , if the sum $m+n$ is even then m and n are both odd or m and n are both even.

For integers m and n , if m and n are neither both odd nor both even, then the sum $m+n$ is not even.

Or

For integers m and n , if one is odd and the other is even, then the sum $m+n$ is odd.

Some tempting but incorrect answers to B from a number of students

(a) Not all isosceles triangles are equilateral.

This is true, but not equivalent to (a). If you draw a Venn diagram, this says that the set of isosceles triangles is not contained in the set of equilateral triangles. But (a) say that the set of equilateral triangles is a subset of the set of isosceles triangles.

(b) If two triangles are not similar, then they are not equilateral.

Of course one of them can be equilateral, so you have to form your statement so that this is clearly and correctly stated. The authors may have *meant* this, but did not say it.

(d) If two angles in a triangle are not complementary, then the triangle is not a right triangle.

The problem here is that literally as stated, one can take two angles and test. So take angle 1 = 30 degree and angle 2 = 90 degrees.

Part C. NAND

Some mathematicians write $A|B$ to mean “A and B are not both true”.

a) Make a truth table for $A|B$.

A	B	$A B$	A and B	$\sim(A \text{ and } B)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

b) Find a formula using only the connectives “and”, “or” and “not” that is equivalent to $A|B$.

$\sim(A \text{ and } B)$