1 Practice questions for midterm

1. Solve the following differential equation

\[ y' = -(y \cos(x) + 1)/\sin(x) \]

as an exact equation and as a linear equation. Verify that the answers agree.

2. Find an integrating factor for the following linear differential equation.

\[ y' = \ln(t)y + e^t \]

3. Which of the following equations are separable?

\[
\begin{align*}
y't + y'y + y + t &= 1 \\
y' + y't + t + 1 &= 1 \\
y' + y + t + 1 &= 0 \\
y'y + t + 1 &= 0
\end{align*}
\]

4. Write down the largest interval on which the following initial value problem is guaranteed to have a unique solution.

\[
\begin{align*}
y' &= -(y \cos(x) + 1)/\sin(x) \\
y(4) &= 1
\end{align*}
\]

5. Consider the following autonomous system

\[ y' = \int_0^y \exp(s^2)ds \]

Write down all the critical point(s), and determine the nature of these critical point(s).

6. Consider the following autonomous system

\[ y' = y^3(y + 1)^2 \]

Write down all the critical point(s), and determine the nature of these critical point(s).
7. Use Euler’s approximation with $n = 1$ to approximate the solution to the following IVP

$$\begin{align*}
y' &= \ln(t + y) \\
y(0) &= 1
\end{align*}$$

at $t = 2$.

Repeat the above with $n = 2$.

8. Consider the following second order differential equation with constant coefficients

$$2y'' + y' - y = 0 \quad (1)$$

Which of the following is a fundamental set for $(1)$? (there is only one correct answer)

(a) $\{e^t, e^{-t}\}$  
(b) $\{e^{t/2}, e^{-t}, e^{t/2} + e^{-t}\}$  
(c) $\{e^{-t}\}$  
(d) $\{e^{t/2} - e^{-t}, e^{-t} - e^{t/2}\}$  
(e) $\{e^{t/2} - e^{-t}, e^{-t} + e^{t/2}\}$

9. Write down a fundamental set to the following second order differential equation with constant coefficients

$$y'' - 6y' + 13 = 0.$$ 

10. Consider the following differential equation

$$t^2 y'' - ty' + y = 0. \quad (2)$$

Let $\{y_1, y_2\}$ be a fundamental set. Suppose that $W(y_1, y_2; 1) = 1$, what is $W(y_1, y_2; t)$?

11. Consider the following second order linear differential equation

$$t^2 y'' + ty' - y = 0. \quad (3)$$

Which of the following is a fundamental set for $(3)$? (there is only one correct answer)

(a) $\{t, 1\}$  
(b) $\{\ln(t), t \ln(t)\}$  
(c) $\left\{t, \frac{t^2 + 1}{t}\right\}$  
(d) $\{t, t^2\}$  
(e) $\{t - 1, 1/t\}$. 

2