Homework 2 for "Algebraic Structures I", Autumn 2010
due Friday, October 29

Problem 1. Classify irreducible modules for sl_2 for

(1) k an arbitrary field of characteristic 0
(2) k a field of positive characteristic p

Problem 2. Show that the functor $U : Lie \to Ass$ that associates to a Lie algebra its universal enveloping algebra is left adjoint to the functor $F : Ass \to Lie$ that sends an associative algebra A to the Lie algebra A^{Lie}.

Problem 3. Let A be a Hopf algebra, and let $\nabla : A \to A \otimes A$ be the comultiplication. An element $a \in A$ is called primitive if $\nabla(a) = a \otimes 1 + 1 \otimes a$. The subspace of all primitive elements is denoted $Prim(A)$. Show that $Prim(U(g)) = g$ for any Lie algebra g over a field k of characteristic 0.

Problem 4. Let x, y be commuting semisimple elements in gl_n. Show that $x + y$ is semisimple. Give an example when this fails if x, y do not commute.