Final for 506, Spring 2009
Wednesday, June 10

You may use you notes and anything we proved in class or homework: just state clearly the fact/theorem you are using. If in doubt, ask me.

Throughout, A is a commutative ring with identity.

Problem 1.[10pt] Let S be a multiplicatively closed set in A, and M be a finitely generated A-module. Show that $S^{-1}(\operatorname{Ann}_A M) = \operatorname{Ann}_{S^{-1}A}(S^{-1}M)$.

Problem 2.[10pt]

(1) Let M, N be flat A-modules. Show that $M \otimes_A N$ is also flat.
(2) Let $0 \longrightarrow M' \longrightarrow M \longrightarrow M'' \longrightarrow 0$ be a short exact sequence of A-modules, and assume that M'' is flat. Show that M is flat if and only if M' is flat.

Problem 3.[10pt] Let p be a prime. Describe the following topological spaces (points, irreducible components and dimension):

(1) $\operatorname{Spec} \mathbb{Z}_{(p)}$,
(2) $\operatorname{Spec} \mathbb{Z}_{(p)}[x]$.

Problem 4.[10pt] Let A be a Noetherian ring, $\mathfrak{p}$ be a prime ideal in A, and $S_\mathfrak{p} = A - \mathfrak{p}$. Let

$$\mathfrak{p}^{(n)} = (\mathfrak{p}^n : S_\mathfrak{p}) = \{a \in A \mid \exists\ s \in S_\mathfrak{p} \text{ such that } as \in \mathfrak{p}^n\}$$

be the n^{th} symbolic power of $\mathfrak{p}$. Show that $\mathfrak{p}^{(n)}$ is the $\mathfrak{p}$-primary component of a primary decomposition of $\mathfrak{p}^n$.

Problem 5.[Bonus 10pt] Show that any ideal in a Dedekind ring can be generated by two elements.