

**Homework 5 for 504, Fall 2018**  
due Wednesday, November 14

This homework is *optional*. Doing it will not earn you any monetary or other tangible rewards. Not doing it will not affect your grade for the course negatively. Yet, doing it will earn you an eternal mathematical blessing and may affect your grade positively, especially if your overall grade is “borderline”.

**Problem 1.** (1). Show that the group with presentation

$$\langle a, b \mid a^4 = b^4 = 1, bab^{-1} = a^{-1} \rangle$$

has order 16.

(2). Show that the group with presentation

$$\langle a, b, c \mid a^{-1}ba = b^2, b^{-1}cb = c^2, c^{-1}ac = a^2 \rangle$$

is trivial.

**Problem 2.** Let  $G$  be a group generated by two elements. Assume further that any element in  $G$  has order dividing 3. Show that  $|G| \leq 27$  and that this estimate cannot be improved.