Problem 1. Let $F$ be a field of characteristic $p > 0$. $F$ is called **perfect** if $F^p = F$. Show that $F$ is perfect if and only if any algebraic extension $L/F$ is separable.

Problem 2. Let $K = \mathbb{Q}(z)$, a field of rational functions on one variable. Consider $f(X) = X^n - z \in K[X]$.
(a) Show that $f(X)$ is irreducible.
(b) Describe the splitting field of $f$.
(c) Determine the Galois group of the splitting field of $X^5 - z$ over $K$.

Problem 3. Determine the Galois group (order, name) of the splitting field of $x^4 - 2$ over $\mathbb{Q}$. Determine all normal subextensions of the splitting field over $\mathbb{Q}$.

Problem 4. Let $L/F$ be a finite Galois extension, and let $f(x)$ be an irreducible polynomial over $F$ of degree 7 which does not have roots in $L$. Show that $f$ is irreducible over $L$.

Problem 5. Let $L$ be the splitting field of the polynomial $x^5 - 4x + 2$ over $\mathbb{Q}$. Prove that $\text{Gal}(L/\mathbb{Q}) \simeq S_5$. 
