Postulates of Euclidean Geometry

Postulates 1–9 of Neutral Geometry.

Postulate 10E (The Euclidean Parallel Postulate). *For each line ℓ and each point A that does not lie on ℓ, there is a unique line that contains A and is parallel to ℓ.*

Postulate 11E (The Euclidean Area Postulate). *For every polygonal region $\mathfrak{R}$, there is a positive real number $S(\mathfrak{R})$ called the **area of** $\mathfrak{R}$, which satisfies the following conditions:*

(*i*) (Area Congruence Property) *If $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are congruent simple regions, then $S(\mathfrak{R}_1) = S(\mathfrak{R}_2)$.*
(*ii*) (Area Addition Property) *If $\mathfrak{R}_1, \ldots, \mathfrak{R}_n$ are nonoverlapping simple regions, then $S(\mathfrak{R}_1 \cup \cdots \cup \mathfrak{R}_n) = S(\mathfrak{R}_1) + \cdots + S(\mathfrak{R}_n)$.*
(*iii*) (Unit Area Property) *If $\mathfrak{R}$ is a square region with sides of length 1, then $S(\mathfrak{R}) = 1$.*

Selected Theorems of Euclidean Geometry

All of the theorems of neutral geometry.

Theorem 10.1 (Converse to the Alternate Interior Angles Theorem). *If two parallel lines are cut by a transversal, then both pairs of alternate interior angles are congruent.*

Corollary 10.2 (Converse to the Corresponding Angles Theorem). *If two parallel lines are cut by a transversal, then all four pairs of corresponding angles are congruent.*

Corollary 10.3 (Converse to the Consecutive Interior Angles Theorem). *If two parallel lines are cut by a transversal, then both pairs of consecutive interior angles are supplementary.*

Lemma 10.4 (Proclus's Lemma). *Suppose ℓ and ℓ' are parallel lines. If t is a line that is distinct from ℓ but intersects ℓ, then t also intersects ℓ'.*

Theorem 10.5. *Suppose ℓ and ℓ' are parallel lines. Then any line that is perpendicular to one of them is perpendicular to both.*

Corollary 10.6. *Suppose ℓ and ℓ' are parallel lines, and m and m' are distinct lines such that $m \perp \ell$ and $m' \perp \ell'$. Then $m \parallel m'$.*

Corollary 10.7 (Converse to the Common Perpendiculars Theorem). *If two lines are parallel, then they have a common perpendicular.*

Theorem 10.8 (Converse to the Equidistance Theorem). *If two lines are parallel, then each one is equidistant from the other.*

Corollary 10.9 (Symmetry of Equidistant Lines). *If ℓ and m are two distinct lines, then ℓ is equidistant from m if and only if m is equidistant from ℓ.*

Theorem 10.10 (Transitivity of Parallelism). *If ℓ, m, and n are distinct lines such that $\ell \parallel m$ and $m \parallel n$, then $\ell \parallel n$.*

Theorem 10.11 (Angle-Sum Theorem for Triangles). *Every triangle has angle sum equal to $180°$.*

Corollary 10.12. *In any triangle, the measure of each exterior angle is equal to the sum of the measures of the two remote interior angles.*

Theorem 10.13 (60-60-60 Theorem). *A triangle has all of its interior angle measures equal to $60°$ if and only if it is equilateral.*

Theorem 10.14 (30-60-90 Theorem). *A triangle has interior angle measures $30°$, $60°$, and $90°$ if and only if it is a right triangle in which the hypotenuse is twice as long as the shortest leg.*

Theorem 10.15 (45-45-90 Theorem). *A triangle has interior angle measures* $45°$, $45°$, *and* $90°$ *if and only if it is an isosceles right triangle.*

Theorem 10.16 (Euclid's Fifth Postulate). *If* ℓ *and* ℓ' *are two lines cut by a transversal* t *in such a way that the measures of two consecutive interior angles add up to less than* $180°$, *then* ℓ *and* ℓ' *intersect on the same side of* t *as those two angles.*

Theorem 10.17 (AAA Construction Theorem). *Suppose* $\overline{AB}$ *is a segment, and* α, β, *and* γ *are three positive real numbers whose sum is* 180. *On each side of* $\overleftrightarrow{AB}$, *there is a point* C *such that* $\triangle ABC$ *has the following angle measures:* $m\angle A = \alpha°$, $m\angle B = \beta°$, *and* $m\angle C = \gamma°$.

Corollary 10.18 (Equilateral Triangle Construction Theorem). *If* $\overline{AB}$ *is any segment, then on each side of* $\overleftrightarrow{AB}$ *there is a point* C *such that* $\triangle ABC$ *is equilateral.*

Theorem 10.19 (Angle-Sum Theorem for Convex Polygons). *In a convex polygon with* n *sides, the angle sum is equal to* $(n-2)\times 180°$.

Corollary 10.20. *In a regular* n*-gon, the measure of each angle is* $\frac{n-2}{n} \times 180°$.

Corollary 10.21 (Exterior Angle Sum for a Convex Polygon). *In any convex polygon, the sum of the measures of the exterior angles (one at each vertex) is* $360°$.

Theorem 10.22 (Angle-Sum Theorem for General Polygons). *If* $\mathfrak{P}$ *is any polygon with* n *sides, the sum of its interior angle measures is* $(n-2)\times 180°$.

Theorem 10.23 (Angle Sum Theorem for Quadrilaterals). *Every convex quadrilateral has an angle sum of* $360°$.

Corollary 10.24. *A quadrilateral is equiangular if and only if it is a rectangle, and it is a regular quadrilateral if and only if it is a square.*

Theorem 10.25. *Every parallelogram has the following properties.*

(*a*) *Each diagonal cuts it into a pair of congruent triangles.*
(*b*) *Both pairs of opposite sides are congruent.*
(*c*) *Both pairs of opposite angles are congruent.*
(*d*) *Its diagonals bisect each other.*

Theorem 10.26. *If a quadrilateral has a pair of opposite sides that are both parallel and congruent, then it is a parallelogram.*

Theorem 10.27. *If a quadrilateral has a pair of opposite sides that are both perpendicular to a third side and congruent, then it is a rectangle.*

Theorem 10.28 (Constructing a Rectangle). *Suppose* a *and* b *are positive real numbers, and* $\overline{AB}$ *is a segment of length* a. *On either side of* $\overleftrightarrow{AB}$, *there exist points* C *and* D *such that* $ABCD$ *is a rectangle with* $AB = CD = a$ *and* $AD = BC = b$.

Corollary 10.29 (Constructing a Square). *If* $\overline{AB}$ *is any segment, then on each side of* $\overleftrightarrow{AB}$ *there are points* C *and* D *such that* $ABCD$ *is a square.*

Theorem 10.30 (Midsegment Theorem). *Any midsegment of a triangle is parallel to the third side and half as long.*

Chapter 11: Area

Lemma 11.1 (Convex Decomposition Lemma). *Suppose* $\mathfrak{P}$ *is a convex polygon, and* $\overline{BC}$ *is a chord of* $\mathfrak{P}$. *Then the two convex polygons* $\mathfrak{P}_1$ *and* $\mathfrak{P}_2$ *described in the polygon splitting theorem (Theorem 8.9) form an admissible decomposition of* $\mathfrak{P}$, *and therefore* $S(\mathfrak{P}) = S(\mathfrak{P}_1) + S(\mathfrak{P}_2)$.

Lemma 11.2. *Suppose* $\mathfrak{P}$ *is a convex polygon,* O *is a point in* $\operatorname{Int}\mathfrak{P}$, *and* $\{B_1, \dots, B_m\}$ *are distinct points on* $\mathfrak{P}$, *ordered in such a way that for each* $i = 1, \dots, m$, *the angle* $\angle B_i O B_{i+1}$ *is proper and contains none of the* B_j*'s in its interior (where we interpret* B_{m+1} *to mean* B_1*). For each* $i = 1, \dots, m$, *let* $\mathfrak{P}_i$ *denote the following set:*

$$\mathfrak{P}_i = \overline{OB_i} \cup \overline{OB_{i+1}} \cup \big(\mathfrak{P} \cap \operatorname{Int}\angle B_i O B_{i+1}\big).$$

Then each $\mathfrak{P}_i$ is a convex polygon, and

$$S(\mathfrak{P}) = S(\mathfrak{P}_1) + \cdots + S(\mathfrak{P}_m). \tag{11.1}$$

Theorem 11.8 (Area of a Rectangle). *The area of a rectangle is the product of the lengths of any two adjacent sides.*

Lemma 11.9 (Area of a Right Triangle). *The area of a right triangle is one-half of the product of the lengths of its legs.*

Theorem 11.10 (Area of a Triangle). *The area of a triangle is equal to one-half the length of any base multiplied by the corresponding height.*

Corollary 11.11 (Triangle Sliding Theorem). *Suppose $\triangle ABC$ and $\triangle A'BC$ are triangles that have a common side $\overline{BC}$, such that A and A' both lie on a line parallel to $\overleftrightarrow{BC}$. Then $S_{\triangle ABC} = S_{\triangle A'BC}$.*

Corollary 11.12 (Triangle Area Proportion Theorem). *Suppose $\triangle ABC$ and $\triangle AB'C'$ are triangles with a common vertex A, such that the points B, C, B', C' are collinear Then*

$$\frac{S_{\triangle ABC}}{S_{\triangle AB'C'}} = \frac{BC}{B'C'}.$$

Theorem 11.13 (Area of a Trapezoid). *The area of a trapezoid is the average of the lengths of the bases multiplied by the height.*

Corollary 11.14 (Area of a Parallelogram). *The area of a parallelogram is the length of any base multiplied by the corresponding height.*

Chapter 12: Similarity

Theorem 12.1 (Transitivity of Similarity of Triangles). *Two triangles that are both similar to a third triangle are similar to each other.*

Theorem 12.2 (The Side-Splitter Theorem). *Suppose $\triangle ABC$ is a triangle, and ℓ is a line parallel to $\overleftrightarrow{BC}$ that intersects $\overline{AB}$ at an interior point D. Then ℓ also intersects $\overline{AC}$ at an interior point E, and the following proportions hold:*

$$\frac{AD}{AB} = \frac{AE}{AC} \qquad \text{and} \qquad \frac{AD}{DB} = \frac{AE}{EC}.$$

Theorem 12.3 (AA Similarity Theorem). *If there is a correspondence between the vertices of two triangles such that two pairs of corresponding angles are congruent, then the triangles are similar under that correspondence.*

Theorem 12.4 (Similar Triangle Construction Theorem). *If $\triangle ABC$ is a triangle and $\overline{DE}$ is any segment, then on each side of $\overleftrightarrow{DE}$, there is a point F such that $\triangle ABC \sim \triangle DEF$.*

Theorem 12.5 (SSS Similarity Theorem). *If $\triangle ABC$ and $\triangle DEF$ are triangles such that $AB/DE = AC/DF = BC/EF$, then $\triangle ABC \sim \triangle DEF$.*

Theorem 12.6 (SAS Similarity Theorem). *If $\triangle ABC$ and $\triangle DEF$ are triangles such that $\angle A \cong \angle D$ and $AB/DE = AC/DF$, then $\triangle ABC \sim \triangle DEF$.*

Theorem 12.7 (Two Transversals Theorem). *Suppose ℓ and ℓ' are parallel lines, and m and n are two distinct transversals to ℓ and ℓ' meeting at a point X that is not on either ℓ or ℓ'. Let M and N be the points where m and n, respectively, meet ℓ; and let M' and N' be the points where they meet ℓ'. Then $\triangle XMN \sim \triangle XM'N'$.*

Theorem 12.8 (Converse to the Side-Splitter Theorem). *Suppose $\triangle ABC$ is a triangle, and D and E are interior points on $\overline{AB}$ and $\overline{AC}$, respectively, such that*

$$\frac{AD}{AB} = \frac{AE}{AC}.$$

Then $\overleftrightarrow{DE}$ is parallel to $\overleftrightarrow{BC}$.

Theorem 12.9 (Angle Bisector Proportionality Theorem). *Suppose $\triangle ABC$ is a triangle and D is a point on $\overline{BC}$ that also lies on the bisector of $\angle BAC$. Then*

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

Theorem 12.10 (Parallel Projection Theorem). *Suppose ℓ, m, n, t, and t' are distinct lines such that $\ell \parallel m \parallel n$; t intersects ℓ, m, and n at A, B, and C, respectively; and t' intersects the same three lines at A', B', and C', respectively. If B is between A and C, then B' is between A' and C', and*

$$\frac{AB}{BC} = \frac{A'B'}{B'C'}.$$

Theorem 12.11 (Menelaus's Theorem). *Let $\triangle ABC$ be a triangle. Suppose D, E, F are points different from A, B, C and lying on such that either two of the points lie on $\triangle ABC$ or none of them do. Then D, E, and F are collinear if and only if*

$$\frac{AD}{DB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FA} = 1.$$

Theorem 12.12 (Ceva's Theorem). *Suppose $\triangle ABC$ is a triangle, and D, E, F are points in the interiors of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively Then the three cevians $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are concurrent if and only if*

$$\frac{AD}{DB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FA} = 1.$$

Theorem 12.13 (Median Concurrence Theorem). *The medians of a triangle are concurrent, and the distance from the point of intersection to each vertex is twice the distance to the midpoint of the opposite side.*

Theorem 12.19 (Triangle Area Scaling Theorem). *If two triangles are similar, then the ratio of their areas is the square of the ratio of their corresponding side lengths; that is, if $\triangle ABC \sim \triangle DEF$ and $AB = r \cdot DE$, then $S_{\triangle ABC} = r^2 \cdot S_{\triangle DEF}$.*

Theorem 12.20 (Quadrilateral Area Scaling Theorem). *If two convex quadrilaterals are similar, then the ratio of their areas is the square of the ratio of their corresponding side lengths.*

Chapter 13: Right triangles

Theorem 13.1 (The Pythagorean Theorem). *Suppose $\triangle ABC$ is a right triangle with right angle at C, and let a, b, and c denote the lengths of the sides opposite A, B, and C, respectively. Then $a^2 + b^2 = c^2$.*

Theorem 13.2 (Converse to the Pythagorean Theorem). *Suppose $\triangle ABC$ is a triangle with side lengths a, b, and c. If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle, and its hypotenuse is the side of length c.*

Theorem 13.3 (Side Lengths of 30-60-90 Triangles). *In a triangle with angle measures $30°$, $60°$, and $90°$, the longer leg is $\sqrt{3}$ times as long as the shorter leg, and the hypotenuse is twice as long as the shorter leg.*

Theorem 13.4 (Side Lengths of 45-45-90 Triangles). *In a triangle with angle measures $45°$, $45°$, and $90°$, the legs are congruent, and the hypotenuse is $\sqrt{2}$ times as long as either leg.*

Theorem 13.5 (Diagonal of a Square). *In a square, each diagonal is $\sqrt{2}$ times as long as each side.*

Theorem 13.6 (SSS Existence Theorem). *Suppose a, b, and c are positive real numbers such that each one is strictly less than the sum of the other two. Then there exists a triangle with side lengths a, b, and c.*

Corollary 13.7 (SSS Construction Theorem). *Suppose a, b, and c are positive real numbers such that each one is strictly less than the sum of the other two, and $\overline{AB}$ is a segment of length c. Then on either side of $\overleftrightarrow{AB}$, there exists a point C such that $\triangle ABC$ has side lengths a, b, and c opposite vertices A, B, and C, respectively.*

Theorem 13.8 (Right Triangle Similarity Theorem). *The altitude to the hypotenuse of a right triangle cuts it into two triangles that are similar to each other and to the original triangle.*

Theorem 13.9 (Right Triangle Proportion Theorem). *In every right triangle, the following proportions hold:*

(a) *The altitude to the hypotenuse is the geometric mean of the projections of the two legs onto the hypotenuse.*

(*b*) *Each leg is the geometric mean of its projection onto the hypotenuse and the whole hypotenuse.*

Theorem 13.12 (The Pythagorean Identity). *If θ is any real number in the interval $[0, 180]$, then $(\sin\theta)^2 + (\cos\theta)^2 = 1$.*

Theorem 13.13 (The Law of Cosines). *Let $\triangle ABC$ be any triangle, and let a, b, and c denote the lengths of the sides opposite A, B, and C, respectively. Then*

$$a^2 + b^2 = c^2 + 2ab\cos\angle C.$$

Theorem 13.14 (Law of Sines). *Let $\triangle ABC$ be any triangle, and let a, b, and c denote the lengths of the sides opposite A, B, and C, respectively. Then*

$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}.$$

Theorem 13.15 (Heron's Formula). *Let $\triangle ABC$ be a triangle, and let a, b, c denote the lengths of the sides opposite A, B, and C, respectively. Then*

$$S_{\triangle ABC} = (s(s-a)(s-b)(s-c))^{\frac{1}{2}}$$

where $s = (a+b+c)/2$ (called the ***semiperimeter of $\triangle ABC$****).*

Theorem 13.16 (Sum Formulas). *Suppose α and β are real numbers such that α, β, and $\alpha+\beta$ are all strictly between $0°$ and $90°$. Then*

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

Corollary 13.17 (Double Angle Formulas). *Suppose α is a real number strictly between $0°$ and $45°$. Then*

$$\sin 2\alpha = 2\sin\alpha\cos\alpha,$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$$

Chapter: 14 **Circles**

Theorem 14.4 (Properties of Secant Lines). *Suppose $\mathfrak{C}$ is a circle and ℓ is a secant line that intersects $\mathfrak{C}$ at A and B. Then every interior point of the chord $\overline{AB}$ is in the interior of $\mathfrak{C}$, and every point of ℓ that is not in $\overline{AB}$ is in the exterior of $\mathfrak{C}$.*

Theorem 14.5 (Properties of Chords). *Suppose $\mathfrak{C}$ is a circle and $\overline{AB}$ is a chord of $\mathfrak{C}$.*

(*a*) *The perpendicular bisector of $\overline{AB}$ passes through the center of $\mathfrak{C}$.*

(*b*) *If $\overline{AB}$ is not a diameter, a radius of $\mathfrak{C}$ is perpendicular to $\overline{AB}$ if and only if it bisects $\overline{AB}$.*

Theorem 14.6 (Line-Circle Theorem). *Suppose $\mathfrak{C}$ is a circle and ℓ is a line that contains a point in the interior of $\mathfrak{C}$. Then ℓ is a secant line for $\mathfrak{C}$, and thus there are exactly two points where ℓ intersects $\mathfrak{C}$.*

Theorem 14.7 (Tangent Line Theorem). *Suppose $\mathfrak{C}$ is a circle, and ℓ is a line that intersects $\mathfrak{C}$ at a point P. Then ℓ is tangent to $\mathfrak{C}$ if and only if ℓ is perpendicular to the radius through P.*

Corollary 14.8. *If $\mathfrak{C}$ is a circle and $A \in \mathfrak{C}$, there is a unique line tangent to $\mathfrak{C}$ at A.*

Theorem 14.9 (Properties of Tangent Lines). *If $\mathfrak{C}$ is a circle and ℓ is a line that is tangent to $\mathfrak{C}$ at P, then every point of ℓ except P lies in the exterior of $\mathfrak{C}$, and every point of $\mathfrak{C}$ except P lies on the same side of ℓ as the center of $\mathfrak{C}$.*

Lemma 14.15. *Any two conjugate arcs have measures adding up to $360°$.*

Theorem 14.16 (Another Thales's Theorem). *Any angle inscribed in a semicircle is a right angle.*

Theorem 14.17 (Converse to Thales's Theorem). *The hypotenuse of a right triangle is a diameter of a circle that contains all three vertices.*

Theorem 14.18 (Existence of Tangent Lines Through an Exterior Point). *Let $\mathfrak{C}$ be a circle, and let A be a point in the exterior of $\mathfrak{C}$. Then there are exactly two distinct tangent lines to $\mathfrak{C}$ containing A. The two points of tangency X and Y are equidistant from A, and the center of $\mathfrak{C}$ lies on the bisector of $\angle XAY$.*

Theorem 14.19 (Inscribed Angle Theorem). *The measure of a proper angle inscribed in a circle is one-half the measure of its intercepted arc.*

Corollary 14.20 (Arc Addition Theorem). *Suppose A, B, and C are three distinct points on a circle $\mathfrak{C}$, and $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are arcs that intersect only at B. Then $m\,\overset{\frown}{ABC} = m\,\overset{\frown}{AB} + m\,\overset{\frown}{BC}$.*

Corollary 14.21 (Intersecting Chords Theorem: Power of a Point). *Suppose $\overline{AB}$ and $\overline{CD}$ are two distinct chords of a circle $\mathfrak{C}$ that intersect at a point $P \in \operatorname{Int}\mathfrak{C}$. Then*

$$(PA)(PB) = (PC)(PD). \tag{14.2}$$

Corollary 14.22 (Intersecting Secants Theorem: Power of a Point). *Suppose two distinct secant lines of a circle $\mathfrak{C}$ intersect at a point P exterior to $\mathfrak{C}$. Let A, B be the points where one of the secants meets $\mathfrak{C}$, and C, D be the points where the other one does. Then*

$$(PA)(PB) = (PC)(PD). \tag{14.3}$$

Theorem (Circumscribed circle theorem). For every triangle there exists a circumscribed cirlce: the circle that contans all three vertices of the triangle. The center of the circumscribed circle is the intersection point of the three perpendicular bisectors of the triangle.

Theorem (Inscribed circle theorem). For every triangle there exists an inscribed cirlce: the circle that is tangent to all three sides of the triangle. The center of the circumscribed circle is the intersection point of the three angle bisectors of the triangle.

Theorem 14.28 (Cyclic Quadrilateral Theorem). *A quadrilateral $ABCD$ is cyclic if and only if it is convex and both pairs of opposite angles are supplementary: $m\angle A + m\angle C = 180°$ and $m\angle B + m\angle D = 180°$.*

Theorem (Concurrence theorems). Let $\triangle ABC$ be a triangle.

1. The medians of $\triangle ABC$ are concurrent.
2. The angle bisectors of $\triangle ABC$ are concurrent.
3. The perpendicular bisectors of $\triangle ABC$ are concurrent.
4. The lines containing the three altitudes of $\triangle ABC$ are concurrent.

Chapter 16: **Compass and Straightedge Constructions**

Construction Problem 16.1 (Equilateral Triangle on a Given Segment). *Given a segment $\overline{AB}$ and a side of $\overleftrightarrow{AB}$, construct a point C on the chosen side such that $\triangle ABC$ is equilateral.*

Construction Problem 16.2 (Copying a Line Segment to a Given Endpoint). *Given a line segment $\overline{AB}$ and a point C, construct a point X such that $\overline{CX} \cong \overline{AB}$.*

Construction Problem 16.3 (Cutting Off a Segment). *Given two segments $\overline{AB}$ and $\overline{CD}$ such that $CD > AB$, construct a point E in the interior of $\overline{CD}$ such that $\overline{CE} \cong \overline{AB}$.*

Construction Problem 16.4 (Bisecting an Angle). *Given a proper angle, construct its bisector.*

Construction Problem 16.5 (Perpendicular Bisector). *Given a segment, construct its perpendicular bisector.*

Construction Problem 16.6 (Perpendicular Through a Point on a Line). *Given a line ℓ and a point $A \in \ell$, construct the line through A and perpendicular to ℓ.*

Construction Problem 16.7 (Perpendicular Through a Point Not on a Line). *Given a line ℓ and a point $A \notin \ell$, construct the line through A and perpendicular to ℓ.*

Construction Problem 16.8 (Triangle with Given Side Lengths). *Given three segments such that the length of the longest is less than the sum of the lengths of the other two, construct a triangle whose sides are congruent to the three given segments.*

Construction Problem 16.9 (Copying a Triangle to a Given Segment). *Given a triangle $\triangle ABC$, a segment $\overline{DE}$ congruent to $\overline{AB}$, and a side of $\overleftrightarrow{DE}$, construct a point F on the given side such that $\triangle DEF \cong \triangle ABC$.*

Construction Problem 16.10 (Copying an Angle to a Given Ray). *Given a proper angle $\angle ab$, a ray $\vec{c}$, and a side of $\overleftrightarrow{c}$, construct the ray $\vec{d}$ with the same endpoint as $\vec{c}$ and lying on the given side of $\overleftrightarrow{c}$ such that $\angle cd \cong \angle ab$.*

Construction Problem 16.11 (Copying a Convex Quadrilateral to a Given Segment). *Given a convex quadrilateral $ABCD$, a segment $\overline{EF}$ congruent to $\overline{AB}$, and a side of $\overleftrightarrow{EF}$, construct points G and H on the given side such that $EFGH \cong ABCD$.*

Construction Problem 16.12 (Rectangle with Given Side Lengths). *Given any two segments $\overline{AB}$ and $\overline{EF}$, and a side of $\overleftrightarrow{AB}$, construct points C and D on the given side such that $ABCD$ is a rectangle with $\overline{BC} \cong \overline{EF}$.*

Construction Problem 16.13 (Square on a Given Segment). *Given a segment $\overline{AB}$ and a side of $\overleftrightarrow{AB}$, construct points C and D on the chosen side such that $ABCD$ is a square.*

Construction Problem 16.14 (Parallel Through a Point Not on a Line). *Given a line ℓ and a point $A \notin \ell$, construct the line through A and parallel to ℓ.*

Construction Problem 16.15 (Cutting a Segment into n Equal Parts). *Given a segment $\overline{AB}$ and an integer $n \geq 2$, construct points $C_1, \ldots, C_{n-1} \in \operatorname{Int} \overline{AB}$ such that $A * C_1 * \cdots * C_{n-1} * B$ and $AC_1 = C_1C_2 = \cdots = C_{n-1}B$.*

Construction Problem 16.16 (Geometric Mean of Two Segments). *Given two segments $\overline{AB}$ and $\overline{CD}$, construct a third segment that is their geometric mean.*

Construction Problem 16.17 (The Golden Ratio). *Given a line segment $\overline{AB}$, construct a point $E \in \operatorname{Int} \overline{AB}$ such that AB/AE is equal to the golden ratio.*

Construction Problem 16.18 (Parallelogram with the Same Area as a Triangle). *Suppose $\triangle ABC$ is a triangle and $\angle rs$ is a proper angle. Construct a parallelogram with the same area as $\triangle ABC$, and with one of its angles congruent to $\angle rs$.*

Construction Problem 16.19 (Rectangle with a Given Area and Edge). *Given a rectangle $ABCD$, a segment $\overline{EF}$, and a side of $\overleftrightarrow{EF}$, construct a new rectangle with the same area as $ABCD$, with $\overline{EF}$ as one of its edges, and with its opposite edge on the given side of $\overleftrightarrow{EF}$.*

Construction Problem 16.20 (Squaring a Rectangle). *Given a rectangle, construct a square with the same area as the rectangle.*

Construction Problem 16.21 (Squaring a Convex Polygon). *Given a convex polygon, construct a square with the same area as the polygon.*

Construction Problem 16.22 (Doubling a Square). *Given a square, construct a new square whose area is twice that of the original one.*

Circle Constructions

Construction Problem 16.23 (Center of a Circle). *Given a circle, construct its center.*

Construction Problem 16.24 (Inscribed Circle). *Given a triangle, construct its inscribed circle.*

Construction Problem 16.25 (Circumscribed Circle). *Given a triangle, construct its circumscribed circle.*

Constructing Regular Polygons

Construction Problem 16.26 (Square Inscribed in a Circle). *Given a circle and a point A on the circle, construct a square inscribed in the circle that has one vertex at A.*

Construction Problem 16.27 (Regular Pentagon Inscribed in a Circle). *Given a circle and a point A on the circle, construct a regular pentagon inscribed in the circle that has one vertex at A.*

Construction Problem 16.28 (Regular Hexagon Inscribed in a Circle). *Given a circle and a point A on the circle, construct a regular hexagon inscribed in the circle that has one vertex at A.*

Construction Problem 16.29 (Equilateral Triangle Inscribed in a Circle). *Given a circle and a point A on the circle, construct an equilateral triangle inscribed in the circle that has one vertex at A.*

Construction Problem 16.30 (Regular Octagon Inscribed in a Circle). *Given a circle and a point A on the circle, construct a regular octagon inscribed in the circle that has one vertex at A.*