GROUP PROJECTS FOR MATH 445A: GEOMETRY FOR TEACHERS, SPRING 2014

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1. General description

Each project consists of a paper and a presentation. The paper will be graded for mathematical clarity, rigor, exposition, correctness, depth, and relevance to the topic. The paper is worth half of the grade which will be split evenly between the group members unless there is a well grounded suspicion that the work was not split evenly. Presentation is worth the other half of the grade; the criteria are again clarity, correctness, and enthusiastic and engaging delivery. Everyone will be graded for their part of the presentation including answering questions which would test the mastery of the material (that is, even if your job is to do a beautiful power point presentation using your group's notes, you should still know the mathematics behind your project). There will also be points given for the group work.

Paper length: between 3 and 10 pages, typed
Presentation: between 25 and 50 minutes - depending on how much time your group requests; schedule TBA

Choosing your project: Submit your project ranking (on the scale from 1 to 7; 1 is the most desirable, 7 the least desirable) by midnight on Monday, April 14, on the discussion board, under the topic with your group name.

2. Projects

2.1. Project 1: Euler's formula and Platonic solids.

(1) Define planar graphs, give examples
(2) Prove Euler's formula for planar graphs: $V - E + F = 2$.
(3) Classify Platonic solids (regular convex polyhedra) using Euler's formula.

Historical notes, pictures and animation would blend organically into the presentation of this project. Sources: find your own, start with internet search.

2.2. Project 2: Pick's formula and application.

(1) Define "lattice polygons"
(2) State and prove "Pick's formula" which computes the area of a lattice polygon by counting the number of lattice points in the interior and on the boundary, give examples. Possible references:
Alexander Bogomolny's web site `http://www.cut-the-note.org/ctk/Pick_proof.shtml`
Planet Math `http://planetmath.org/ProofOfPicksTheorem`
(3) Application (this is an old Putnam problem): prove that any convex pentagon whose vertices have integer coordinates must have area at least $\frac{5}{2}$.

Date: Spring 2014.

2.3. **Project 3: Apollonian gasket and the Descartes' theorem: the fractal that inspires mathematical poetry.**

(1) Describe the construction of the Apollonian gasket (you can mention and show the Serpinski triangle as well if you'd like)
(2) State and prove the Descartes' theorem on four "kissing" circles. Explain the relevance to the Apollonian gasket.

Some references:

(1) Jeffrey C. Lagarias, Colin L. Mallows and Allan R. Wilks, *Beyond the Descartes Circle Theorem*, The American Mathematical Monthly Vol. 109, No. 4 (Apr., 2002), pp. 338-361 Published by: Mathematical Association of America, Article Stable `http://www.jstor.org/stable/2695498`
(2) D. Mackenzie, *A Tisket, a Tasket, an Apollonian Gasket*, American Scientist, V. 98, no. 1, (2010) `http://www.americanscientist.org/issues/pub/2010/1/a-tisket-a-tasket-apollonian-gasket/5`

This is another potent project for pictures and animation.

2.4. **Project 4: Compass and straight edge constructions I: means.**

(1) Define compass and straight edge constructions.
(2) Define three different "means" for two numbers: arithmetic mean, geometric mean and harmonic mean. Give examples and state the AM-GM-HM inequality.
(3) Prove that given segments of length a, b, the segments of length equal to any of the three means can be constructed.
(4) Application: Construct a square with the same area as a given rectangle.

Much of this material can be found in [1, ch.16]. You don't have to reprove all the constructions done in the book which are needed for the constructions in your project - just state them.

2.5. **Project 5: Compass and straight edge constructions II: golden ratio and regular polygons.**

(1) Show how to construct the golden ratio and the pentagon with a compass and a straight edge.
(2) Show how to construct a regular hexagon with a compass and a straight edge.

This material is in [1, ch.16].

2.6. **Project 6: Congruences for quadrilaterals.** This is Exercise 10K from [1]. Each of the following is a shorthand notation for a possible congruence theorem for convex quadrilaterals: SASSS, SAASS, ASASA, AAASA, SAAAS, ASASS, ASSAS, ASAAS. Try to decide which ones represent valid congruence theorems in Euclidean geometry. When possible, give proofs of those which are valid, and descriptions of counterexamples for those which are not.

2.7. **Project 7: Special cevians in a triangle.** This is a project about various special lines in a triangle.

(1) Define cevians and state and prove Ceva's theorem (Theorem 12.12 in [1]).
(2) Define medians, angle bisectors, perpendicular bisectors and altitudes of a triangle.
(3) Prove that the medians of any triangle are concurrent.

(4) Prove that the angle bisectors of any triangle are concurrent.
(5) Prove that the perpendicular bisectors of any triangle are concurrent.
(6) Prove that the altitudes of any triangle are concurrent.

References

[1] J. Lee, *Axiomatic Geometry*, AMS (2013)
[2] J. Sally, P. Sally, *Geometry: A Guide for Teachers*, MSRI Mathematical Circles Library, AMS and MSRI, (2011).