NAME ______________________________ STUDENT NUMBER __________

Practice Final

Math 445A: Geometry for teachers

June 1, 2014

Problem	Total Points	Score
1		
2		
3		
4		
5		
6		
7		
8		
Total		

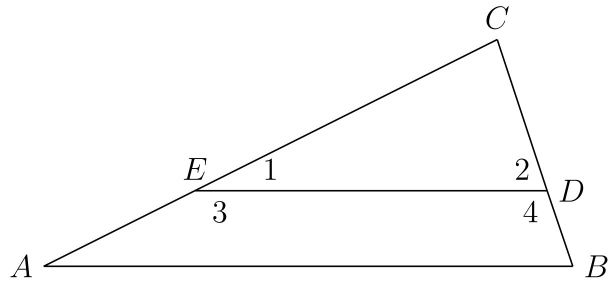
- You may use the lists of postulates and theorems distributed in class and two-sided page of your own notes prepared for the final.

- No other notes, books, or electronic devices. Please turn off your cell phone.

- Show all your work to get full credit. Write your solutions on the pages provided. Use backs for scratch paper if you need it.

- Read instructions for each problem CAREFULLY.

(1) The statements below refer to the following diagram. In this diagram, we are given only that $\triangle ABC$ is a triangle, and D and E are points such that $A * E * C$ and $B * D * C$. The angles $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are defined as shown in the diagram.



For each of the statements below, circle one of the following answers, both, or neither (2 points for each correct answer):

Euclidean: The statement is *true in Euclidean* geometry
Hyperbolic: The statement is *true in Hyperbolic* geometry

(a) **Euclidean** **Hyperbolic** If $\overleftrightarrow{ED} || \overleftrightarrow{AB}$, then $\triangle ABC \sim \triangle EDC$

(b) **Euclidean** **Hyperbolic** If $\angle 3$ and $\angle A$ are supplementary, then $ABDE$ is a trapezoid

(c) **Euclidean** **Hyperbolic** If $m\angle 3 + m\angle A < 180^o$, then $\overleftrightarrow{ED}$ and $\overleftrightarrow{AB}$ are not parallel

(d) **Euclidean** **Hyperbolic** $m\angle 3 > m\angle C + m\angle 2$.

(e) **Euclidean** **Hyperbolic** $m\angle 3 > m\angle 1$.

(f) **Euclidean** **Hyperbolic** $m\angle 4 + m\angle 2 = 180^o$.

(g) **Euclidean** **Hyperbolic** $m\angle C + m\angle 1 + m\angle 2 \leq 180^o$.

(h) **Euclidean** **Hyperbolic** If $|CE| < |CD|$, then $m\angle 1 > m\angle 2$.

(i) **Euclidean** **Hyperbolic** If there is a line ℓ such that $\ell \perp \overleftrightarrow{ED}$ and $\ell \perp \overleftrightarrow{AB}$ then $\overleftrightarrow{ED} || \overleftrightarrow{AB}$.

(2) (a) (2pts) Define the interior of a convex polygon.

(b) (8pts) Give a proof of the following fact in *Neutral Geometry*: if a quadrilateral is convex, then its diagonals intersect at a point that is in the interior of both diagonals and the quadrilateral.

You may use any theorems that come before Theorem 9.4.

(3) (10pts) Let ABC be a triangle and let AD, BE and CF be three concurrent cevians interseting at the point G. Prove that

$$\frac{S_{\triangle ABG}}{S_{\triangle CBG}} = \frac{AF}{FC}$$

(4) (10pts) Prove the "Height Scaling Theorem". You may use any theorem that comes prior to 12.17.

Theorem. *If two triangles are similar, their corresponding heights have the same ratio as their corresponding sides.*

(5) (a) State Pick's formula for the area of a lattice polygon.

(b) Prove Pick's formula for a right triangle with legs parallel to the lattice grid.

(6) (a) (2pts) Define a *cyclic* polygon.

(b) (8pts) Prove that a convex quadrilateral ABCD is cyclic if and only if $m\angle A + m\angle C = m\angle B + m\angle D = 180^o$. You may use anything that comes prior to 14.28.

(7) (10pts) Prove existence of an inscribed circle in Euclidean geometry: for any triangle ABC there exists a circle tangent to all three sides of ABC.
You may use any of the theorems on the list except for the "Inscribed Circle Theorem".

(8) (a) (5pts) Define a planar graph. Define a connected graph. State Euler's formula for connected planar graphs.

(b) (5 pts) Let Γ be a connected planar graph which is not a tree with E edges and F faces. Prove the following inequality:

$$2E \geq 3F$$

(c) (10 pts) Prove that a graph formed by five vertices, five sides and five diagonals of a pentagon is not planar.