HOMEWORK 8 FOR MATH 445A, SPRING 2014

DUE WEDNESDAY, MAY 28

Problem 1. Give yet another proof of the Pythagorean theorem: 13A.

Problem 2. Prove the converse to the Pythagorean theorem: 13C.

Definition 1. The *circumscribed circle* of a triangle ABC is a circle that contains all three of its vertices. The radius of the circumscribed circle is often denoted by R.

Remark 2. The center O of the circumscribed circle to $\triangle ABC$ is the intersection point of the three perpendicular bisectors. We have $R = AO = BO = CO$. Note that it was shown by the Euclid's Unpaid Interns team on Friday that the perpendicular bisectors are concurrent, and, hence, the point O exists. The circumscribed circle is also unique for any given triangle ABC.

Caution: The center of the circumscribed circle can be **outside** of the triangle.

Problem 3. (This is a variation of 13H) Let $\triangle ABC$ be a triangle, let $a = |BC|, b = |AC|, c = |AB|$, and let R be the radius of the circumscribed circle. The *Law of Sines* has the following extended form:

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C} = 2R.$$

Prove this formula under the assumption that all angles of $\triangle ABC$ are acute. **You may use (without proving it)** that in that case the circumcenter O lies in the **interior** of the triangle $\triangle ABC$.

Problem 4. Prove the "Sine area" formula: let ABC be a triangle, let $a = |BC|$, and $b = |AC|$. Then

$$S_{\triangle ABC} = \frac{ab \sin \angle C}{2}.$$

Problem 5. 14B

Problem 6. 14I

Date: May 21, 2014.