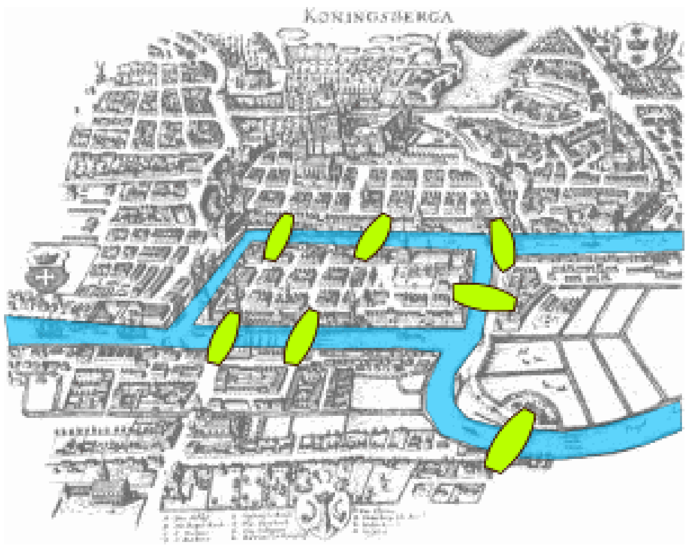
HOMEWORK 7 FOR MATH 445A, SPRING 2014

DUE WEDNESDAY, MAY 21

Problem 1. In the town of Königsburg there are several canals crossed by 7 bridges, exactly as shown in the picture. Is it possible to design a walk through the city which crosses each exactly once?



Problem 2. Let f be a chosen coordinate function on a line ℓ. Let A, B, C be three distinct points on ℓ such that the coordinates $f(A), f(B), f(C)$ are all integers. Prove that at least one of the segments $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ has a midpoint with an integer coordinate.

Problem 3. Let $\triangle ABC$ be a triangle. Prove that the intersection point of the three angle bisectors of $\triangle ABC$ is equidistant from all three sides of the triangle.

Definition 1. The intersection point of the angle bisectors is called the *incenter* of $\triangle ABC$ and is denoted by I. The distance from the incenter I to any of the sides is denoted by r. The circle with the center I and radius r is *tangent* to all three sides and is called the *inscribed circle* of the triangle.

Remark 2. Unlike the *circumcenter* which we will encounter later, the *incenter* always lies inside the triangle.

Problem 4. Let $\triangle ABC$ be a triangle with $|AB| = c$, $|BC| = a$, $|AC| = b$. Let $p = \frac{a+b+c}{2}$ be half of the perimeter, and let r be the radius of the inscribed circle. Prove the following formula for the area:

$$S_{\triangle ABC} = pr$$