Name ______________________ Student Number ____________

Practice Final

Math 445A: Geometry for teachers

June 4, 2013

Problem	Total Points	Score
1		
2		
3		
4		
5		
6		
7		
8		
Total		

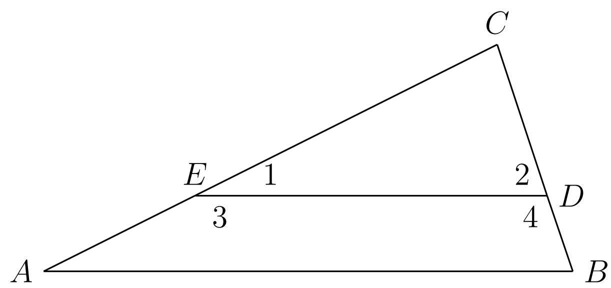
- You may use the lists of postulates and theorems distributed in class and two-sided page of your own notes prepared for the final.

- No other notes, books, or electronic devices. Please turn off your cell phone.

- Show all your work to get full credit. Write your solutions on the pages provided. Use backs for scratch paper if you need it.

- Read instructions for each problem CAREFULLY.

(1) The statements below refer to the following diagram. In this diagram, we are given only that $\triangle ABC$ is a triangle, and D and E are points such that $A * E * C$ and $B * D * C$. The angles $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are defined as shown in the diagram.



For each of the statements below, circle one of the following answers, both, or neither (2 points for each correct answer):

Euclidean: The statement is *true in Euclidean* geometry
Hyperbolic: The statement is *true in Hyperbolic* geometry

(a) **Euclidean** **Hyperbolic** If $\overleftrightarrow{ED} || \overleftrightarrow{AB}$, then $\triangle ABC \sim \triangle EDC$

(b) **Euclidean** **Hyperbolic** If $\angle 3$ and $\angle A$ are supplementary, then $ABDE$ is a trapezoid

(c) **Euclidean** **Hyperbolic** If $m\angle 3 + m\angle A < 180^o$, then $\overleftrightarrow{ED}$ and $\overleftrightarrow{AB}$ are not parallel

(d) **Euclidean** **Hyperbolic** $m\angle 3 > m\angle C + m\angle 2$.

(e) **Euclidean** **Hyperbolic** $m\angle 3 > m\angle 1$.

(f) **Euclidean** **Hyperbolic** $m\angle 4 + m\angle 2 = 180^o$.

(g) **Euclidean** **Hyperbolic** $m\angle C + m\angle 1 + m\angle 2 \leq 180^o$.

(h) **Euclidean** **Hyperbolic** If $|CE| < |CD|$, then $m\angle 1 > m\angle 2$.

(i) **Euclidean** **Hyperbolic** If there is a line ℓ such that $\ell \perp \overleftrightarrow{ED}$ and $\ell \perp \overleftrightarrow{AB}$ then $\overleftrightarrow{ED} || \overleftrightarrow{AB}$.

(2) (a) (2pts) Define the interior of a convex polygon.

(b) (8pts) Give a proof of the following fact in *Neutral Geometry*: if a quadrilateral is convex, then its diagonals intersect at a point that is in the interior of both diagonals and the quadrilateral.

You may use any theorems that come before Theorem 9.4.

(3) (10pts) Prove the "Angle Bisector Proportion theorem". You may use anything prior to Theorem 12.9.

Theorem. *Let ABC be a triangle, and D be the point where the angle bisector of $\angle ABC$ meets AC. Then*

$$\frac{AB}{BC} = \frac{AD}{DC}.$$

(4) (10pts) Prove the "Height Scaling Theorem". You may use any theorem that comes prior to 12.17.

Theorem. *If two triangles are similar, their corresponding heights have the same ratio as their corresponding sides.*

(5) (a) State Pick's formula for the area of a lattice polygon.

(b) Prove Pick's formula for a right triangle with legs parallel to the lattice grid.

(6) (10pts) Solve the following construction problem in Euclidean geometry, and prove that your solution is correct. You may use any postulates, definitions, theorems, and constructions in Euclidean geometry from Chapters 3-13 and 16 that come prior to Construction Problem 16.6.

Construction Problem: Given a line ℓ and a point A on ℓ, construct a line through A and perpendicular to ℓ.

(7) (10pts) Prove existence of a circumcircle in Euclidean geometry. You may use any of the theorems on the list except for the "Circumscribed Circle Theorem".

Theorem. *For any triangle ABC there exists a circle that contains all three vertices A, B, and C.*

(8) (a) (5pts) Define a planar graph. State Euler's formula for planar graphs.

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(b) (5 pts) Let Γ be a planar graph which is not a tree with E edges and F faces. Prove the following inequality:

$$2E \geq 3F$$

(c) (10 pts) Prove that a graph formed by five vertices, five sides and five diagonals of a pentagon is not planar.