HOMEWORK 8 FOR MATH 445A, SPRING 2013

DUE WEDNESDAY, MAY 29

Problem 1. 12L

Problem 2. 13C

Problem 3. Let $\triangle ABC$ be a triangle.

(1) Prove that the perpendicular bisectors to the three sides of $\triangle ABC$ are concurrent.
(2) Let O be the intersection point of the perpendicular bisectors. Show that the point O is equidistant from all three vertices of the triangle $\triangle ABC$. That is, $AO = BO = CO$.

Definition 1. The *circumscribed circle* of a triangle ABC is a circle that contains all three of its vertices. The point O constructed in Problem 2 is the center of the circumscribed circle and is called the *circumcenter*. The radius of the circumscribed circle is often denoted by R.

Remark 2. In Problem 2 you have shown that the circumscribed circle exists: indeed, this is the circle with the center at the point O and radius $R = AO = BO = CO$. The circumscribed circle is also unique for any given triangle ABC.

Caution: The center of the circumscribed circle can be **outside** of the triangle.

Problem 4. (This is a variation of 13H) Let $\triangle ABC$ be a triangle, let $a = |BC|, b = |AC|, c = |AB|$, and let R be the radius of the circumscribed circle. The *Law of Sines* has the following extended form:

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C} = 2R.$$

Prove this formula under the assumption that all angles of $\triangle ABC$ are acute. **You may use (without proving it)** that in that case the circumcenter O lies in the **interior** of the triangle $\triangle ABC$.

Problem 5. Prove the "Sine area" formula: let ABC be a triangle, let $a = |BC|$, and $b = |AC|$. Then

$$S_{\triangle ABC} = \frac{ab \sin \angle C}{2}.$$

Bonus. 12K

Date: May 20, 2013.