Axioms of Incidence Geometry

Incidence Axiom 1. *There exist at least three distinct noncollinear points.*

Incidence Axiom 2. *Given any two distinct points, there is at least one line that contains both of them.*

Incidence Axiom 3. *Given any two distinct points, there is at most one line that contains both of them.*

Incidence Axiom 4. *Given any line, there are at least two distinct points that lie on it.*

Parallel Postulates

The Elliptic Parallel Postulate. For each line ℓ and each point A that does not lie on ℓ, there are no lines that contain A and are parallel to ℓ.

The Euclidean Parallel Postulate. For each line ℓ and each point A that does not lie on ℓ, there is one and only one line that contains A and is parallel to ℓ.

The Hyperbolic Parallel Postulate. For each line ℓ and each point A that does not lie on ℓ, there are at least two distinct lines that contain A and are parallel to ℓ.

Theorems of Incidence Geometry

Theorem 2.25. *Given any point A, there exists another point that is distinct from A.*

Theorem 2.26. *Given any point, there exists a line that contains it.*

Corollary 2.27. *If A and B are points (not necessarily distinct), there is a line that contains both of them.*

Theorem 2.28. *If ℓ is a line and A and B are two distinct points on ℓ, then $\overleftrightarrow{AB} = \ell$.*

Theorem 2.29. *If A and B are distinct points, and C is any point that does not lie on $\overleftrightarrow{AB}$, then A, B, and C are noncollinear.*

Theorem 2.30. *If A, B, and C are noncollinear points, then A and B are distinct, and C does not lie on $\overleftrightarrow{AB}$.*

Corollary 2.31. *If A, B, and C are noncollinear points, then A, B, and C are all distinct. Moreover, A does not lie on $\overleftrightarrow{BC}$, B does not lie on $\overleftrightarrow{AC}$, and C does not lie on $\overleftrightarrow{AB}$.*

Theorem 2.32. *Given a line ℓ and a point A that lies on ℓ, there exists a point B that lies on ℓ and is distinct from A.*

Theorem 2.33. *Given any line, there exists a point that does not lie on it.*

Theorem 2.34. *Given two distinct points A and B, there exists a point C such that A, B, and C are noncollinear.*

Theorem 2.35. *Given any point A, there exist points B and C such that A, B, and C are noncollinear.*

Theorem 2.36. *Given two distinct points A and B, there exists a line that contains A but not B.*

Theorem 2.37. *Given any point, there exists a line that does not contain it.*

Theorem 2.38. *If A, B, and C are noncollinear points, then $\overleftrightarrow{AB} \neq \overleftrightarrow{AC}$.*

Corollary 2.39. *If A, B, and C are noncollinear points, then $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{BC}$ are all distinct.*

Theorem 2.40. *If A, B, and C are collinear points, and neither B nor C is equal to A, then $\overleftrightarrow{AB} = \overleftrightarrow{AC}$.*

Theorem 2.41. *Given two distinct, nonparallel lines, there exists a unique point that lies on both of them.*

Theorem 2.42. *Given any point, there are at least two distinct lines that contain it.*