NAME _____ STUDENT NUMBER _____

Practice MIDTERM

Math 444A: Geometry for teachers

February 8, 2013

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You may use the distributed lists of axioms and theorems and one-sided page of your own notes prepared for the midterm.

- No other notes, books, or electronic devices. Please turn off your cell phone.

- Show all your work to get full credit. Write you solutions on the pages provided. Use backs for scratch paper if you need it.

- Read instructions for each problem CAREFULLY.

- There are five problems total, each problem is worth 10 points.

(1) This is a multiple choice question. Just circle the right answer, no justification necessary. Correct answer is worth 2 points, no answer 0 points, incorrect answer (-1) point.

All: The statement is *true in all* models of incidence geometry
Some: The statement is *true in some* models of incidence geometry but not all
None: The statement is *false in all* models of incidence geometry

(a) **All** **Some** **None** If P and Q are distinct points that lie on a line ℓ then there exists another point R distinct from P and Q, that also lies on ℓ.

(b) **All** **Some** **None** Every point lies on at least two distinct lines.

(c) **All** **Some** **None** There exists a line that contains exactly one point.

(d) **All** **Some** **None** There exists a line that contains exactly two points

(e) **All** **Some** **None** For any point A there exists at least one line ℓ such that A does not lie on ℓ.

(2) (a) (2pts) Define what it means for a given statement to be independent of the four axioms of incidence geometry.

(b) Using models, show that the following statements are independent of the axioms of incidence geometry:

(i) (4pts) The Elliptic parallel postulate

(ii) (4pts) Given any point, there are at least three distinct lines that contain it

(3) For each of the following statements in neutral geometry, write the countrapositive, the converse, the negation, or rewrite in symbolic form, as requested. (Note that not all statements are necessarily true.)

(a) (2pts) There is a line ℓ that contains no points.
Symbolic form:

(b) (2pts) If the points A, B, and C all lie on a line ℓ, then they are collinear
Contrapositive:

(c) (2pts) For any two distinct points A, B, there exists a line ℓ that contains both A and B.
Negation:

(d) (2pts) If ℓ and m are distinct lines, then there are two distinct points A and B such that A lies on ℓ and B lies on m.
Converse:

(e) (2pts) There is a line ℓ containing no points.
Negation:

(4) Let A, B be two distinct points on the plane.

(a) (3pts) Give definition of the segment $\overline{AB}$.

(b) (7pts) Prove Lemma 3.17: *For any two distinct points A, B,* $\overline{AB} \subset \overleftrightarrow{AB}$.

You may use any fact (a theorem, a lemma, or a corollary) from the list of theorems in Chapter 3 with comes before Lemma 3.17.

(5) (10pts) Prove the "Ruler placement theorem": Suppose ℓ is a line and A, B are two distinct points on ℓ. There exists a coordinate function $f : \ell \to \mathbb{R}$ such that $f(A) = 0$ and $f(B) > 0$.

You may use any fact (a theorem, a lemma, or a corollary) from the list of theorems in Chapter 3 with comes before Theorem 3.5.