NAME _____ STUDENT NUMBER _____

Practice Final

Math 444A: Geometry for teachers

March 11, 2013

Problem	Total Points	Score
1	20	
2	22	
3	10	
4	16	
5	10	
6	16	
7	14	
8	12	
Total	120	

- You may use the distributed lists of axioms and theorems and two-sided page of your own notes prepared for the midterm.

- No other notes, books, or electronic devices. Please turn off your cell phone.

- Show all your work to get full credit. Write you solutions on the pages provided. Use backs for scratch paper if you need it.

- Read instructions for each problem CAREFULLY.

(1) This is a multiple choice question. Just circle the right answer, no justification necessary. Correct answer is worth 2 points, no answer 0 points, incorrect answer (-1) point.

All: The statement is *true in all* models of incidence geometry
Some: The statement is *true in some* models of incidence geometry but not all
None: The statement is *false in all* models of incidence geometry

(a) **All** **Some** **None** If P and Q are distinct points that lie on a line ℓ then there exists another point R distinct from P and Q, that also lies on ℓ.

(b) **All** **Some** **None** Every point lies on at least two distinct lines.

(c) **All** **Some** **None** There exists a line that contains exactly one point.

(d) **All** **Some** **None** There exists a line that contains exactly two points.

(e) **All** **Some** **None** For any point A there exists at least one line ℓ such that A does not lie on ℓ.

(f) **All** **Some** **None** If A and B are two distinct points then there exists at least one line ℓ such that neither A nor B lies on ℓ

(g) **All** **Some** **None** For any point A there exists a line ℓ such that A lies on ℓ.

(h) **All** **Some** **None** For any point A there exist two distinct lines such that A does not lie on either one of them.

(i) **All** **Some** **None** Every line has finitely many points.

(j) **All** **Some** **None** There exists a line that contains no points.

(2) For each of the following statements, answer the following question: Can the statement be proved within the framework of *neutral* geometry; that is, assuming postulates 1-9, and using definitions and theorems of neutral geometry. Just circle the right answer, no justification necessary.

Correct answer is worth 2 points, no answer 0 points, incorrect answer (-1) point.

(a) **Yes** **No** The sum of the measures of three angles of a triangle is 180^0.

(b) **Yes** **No** If there is a correspondence between the vertices of two triangles such that two angles and a side of one triangle are congruent to the corresponding angles and side of the other, then the two triangles are congruent.

(c) **Yes** **No** If $\overrightarrow{a} * \overrightarrow{b} * \overrightarrow{c}$ then $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ belong to the same half rotation $HR(\overrightarrow{r}, P)$ for some ray $\overrightarrow{r}$ and some point P not on $\overrightarrow{r}$.

(d) **Yes** **No** If the angles are adjacent then they are supplementary.

(e) **Yes** **No** If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$, $\overrightarrow{d}$ are rays such that $\overrightarrow{a} * \overrightarrow{b} * \overrightarrow{c}$ and $\overrightarrow{b} * \overrightarrow{c} * \overrightarrow{d}$, then $\overrightarrow{a} * \overrightarrow{b} * \overrightarrow{d}$.

(f) **Yes** **No** If A and B are distinct points, and M is a point such that $AM = MB$, then M is the midpoint of the segment $\overline{AB}$.

(g) **Yes** **No** If $\overleftrightarrow{AB}$ intersects ℓ then A, B are on opposite sides of ℓ.

(h) **Yes** **No** If $A * B * C$, then A, B, and C are collinear.

(i) **Yes** **No** Any angle is convex.

(j) **Yes** **No** If $|AB| + |BC| = |AC|$ then $A * B * C$.

(k) **Yes** **No** Given any two distinct lines, there is at most one point that belongs to both of them.

(3) (10pts) Describe the "Fano plane" model of incidence geometry (draw a picture or give a description in words). Give at least five different reasons, *with justification*, why the Fano plane is not a model for neutral geometry (that is, state five different postulates/theorems/lemma/corollaries of neutral geometry violated by the Fano plane model).

(4) For each of the following statements in neutral geometry, write the countrapositive, the converse, the negation, or rewrite in symbolic form, as requested. For a negation, simplify your answer so that it does not contain a double negative.

(a) (2pts) If C is an interior point of $\overline{AB}$ then $|AC| < |AB|$.
Converse:

(b) (2pts) If the points A, B, and C all lie on a line ℓ, then they are collinear.
Contrapositive:

(c) (2pts) For any two distinct points A, B, there exists a line ℓ that contains both A and B.
Negation:

(d) (2pts) If $\angle ab$ and $\angle bc$ form a linear pair, then $m\angle ab + m\angle bc = 180^0$
Contrapositive:

(e) (2pts) If M is a midpoint of $\overline{AB}$ then $|AM| = |MB|$
Converse:

(f) (2pts) For some pair of points A, B that do not lie on ℓ, the segment $\overline{AB}$ intersects ℓ.
Negation:

(g) (2pts) If A, B, C are distinct collinear points then $A*B*C$ or $A*C*B$ or $B*A*C$.
Negation:

(h) (2pts) If there is a correspondence between the vertices of two triangles such that all three sides of one triangle are congruent to the corresponding sides of the other triangle, then the triangles are congruent under that correspondence.

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Contrapositive:

(5) (10pts) Prove the following theorem in incidence geometry:

Theorem 2.42. *Given any point, there are at least two distinct lines that contain it.*

You may use any axioms, definitions and theorems of *incidence* geometry *except* for Theorem 2.42 itself.

(6) (a) (2pts) Define what it means for two angles to be adjacent.

(b) (2pts) Define what it means for two angles to be supplementary.

(c) (2pts) Define what it means for two angles to form a linear pair

(d) (10pts) Prove the "Partial converse to the Linear Pair Theorem":

Theorem 4.16. *If two adjacent angles are supplementary then they form a linear pair.*

You may use any fact (a theorem, a lemma, or a corollary) from the list of theorems in Chapters 3-4 with comes before Theorem 4.16.

(7) (a) (2pts) Define a bisector of a proper angle $\angle ab$.

(b) (2pts) Define a line perpendicular to a given line.

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(c) (10pts) Prove "The Four right angles theorem":

Theorem 4.29. *If ℓ and m are perpendicular lines then they form four right angles.*

You may use any fact (a theorem, a lemma, or a corollary) from the list of theorems in Chapters 3-4 with comes before Theorem 4.29.

(8) (12pts) Prove Theorem 5.16 (Scalene inequality):

Theorem 5.16. *Let $\triangle ABC$ be a triangle. Then $|AB| > |BC|$ if and only if $m\angle C > m\angle A$.*

You may use any fact (a theorem, a lemma, or a corollary) from the list of theorems in Chapters 3-5 with comes before Theorem 5.16.