NAME _____ STUDENT NUMBER _____

MIDTERM

Math 444A: Geometry for teachers

February 15, 2013

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

- You may use the distributed lists of axioms and theorems and one-sided page of your own notes prepared for the midterm.

- No other notes, books, or electronic devices. Please turn off your cell phone. Individual work only!

- Show all your work to get full credit. Write you solutions on the pages provided. Use backs for scratch paper if you need it.

- Read instructions for each problem CAREFULLY.

- There are four problems total, each problem is worth 10 points.

(1) This is a multiple choice question. Just circle the right answer, no justification necessary. Correct answer is worth 2 points, no answer 0 points, incorrect answer (-1) point.

All: The statement is *true in all* models of incidence geometry
Some: The statement is *true in some* models of incidence geometry but not all
None: The statement is *false in all* models of incidence geometry

(a) **All** **Some** **None** If A and B are two distinct points then there exists at least one line ℓ such that neither A nor B lies on ℓ

(b) **All** **Some** **None** For any point A there exists a line ℓ such that A lies on ℓ.

(c) **All** **Some** **None** For any point A there exist two distinct lines such that A does not lie on either one of them.

(d) **All** **Some** **None** Every line has finitely many points.

(e) **All** **Some** **None** There exists a line that contains no points.

(2) (a) (2pts) Define what it means for a given statement to be independent of the four axioms of incidence geometry.

(b) Using models, show that the following statements are independent of the axioms of incidence geometry:

(i) (4pts) The Hyperbolic parallel postulate.

(ii) (4pts) Given any line, there are at least two distinct points that do not lie on it.

(3) For each of the following statements in neutral geometry, write the countrapositive, the converse, the negation, or rewrite in symbolic form, as requested. (Note that not all statements are necessarily true.)

(a) (2pts) There exist two distinct lines such that no point lies on both of them.
Negation:

(b) (2pts) If the points A, B, and C satisfy the relation $A * B * C$ then $|AC| = |AB| + |BC|$.
Contrapositive:

(c) (2pts) If the points A, B, and C satisfy the relation $A * B * C$ then $|AC| = |AB| + |BC|$.
Converse:

(d) (2pts) Given any two distinct points there exists at most one line that contains both of them.
Symbolic form:

(e) (2pts) Given any two distinct points there exists at least one line that contains both of them.
Negation:

(4) (a) (2pts) Define what it means for a point B to lie *between points A and C*, denoted $A * B * C$.

(b) (7pts) Prove:

Theorem [Betweenness Theorem for points.] Let A, B, C be points. If $A * B * C$ then $|AC| = |AB| + |BC|$.

You may use any fact from the list of theorems in Chapter 3 with a number less than 3.8.

(c) (1pt) Is the converse to this theorem true in neutral geometry? No justification required, just answer yes or no.

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