MATH 342 Lin Alg II Winter 2004
Practice Midterm **Name:** ____________________

1. Let $A = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}$

   (a) Find eigenvalues and eigenvectors of A.

   (b) Consider a discrete dynamical system $x_{k+1} = Ax_k$. Classify the origin as an attractor, repeller or a saddle point of this dynamical system.

   Let x_0 be the initial state of the dynamical system defined above. Compute the state x_{100} of the system for

   1. $x_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$
   2. $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
   3. $x_0 = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

   (d) What is the direction of the greatest repulsion and greatest attraction of the dynamical system above? Estimate the long term growth rate of x_k.

2. Let $T : \mathbf{R^3} \to \mathbf{R^3}$ be a linear transformation defined by the matrix $A = \begin{bmatrix} -1 & 0 & -3 \\ 3 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix}$, $T(x) = Ax$. Determine whether there exists a basis of $\mathbf{R^3}$ relative to which the matrix of T is diagonal.

3. Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 3 & 2 & 3 \\ -1 & -1 & 1 \end{bmatrix}$. Diagonalize A if possible.

4. Let $T : \mathbf{P^3} \to \mathbf{P^2}$ be a linear transformation given by the differential: for a polynomial $p(t)$,

$$T(p(t)) = p'(t).$$

   Compute the matrix of this linear transformation relative to the bases $< 1, t, t^2 >$ of $\mathbf{P^2}$ and $< 1, t, t^2, t^3 >$ of $\mathbf{P^3}$.

5. Let $T : \mathbf{R^3} \to \mathbf{R^3}$ be a linear transformation defined by the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ 3 & 2 & 3 \\ -1 & -1 & 1 \end{bmatrix}$.

   and let $\mathcal{B} =< b_1, b_2, b_3 >$ where $b_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $b_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Compute the matrix of T relative to the basis $\mathcal{B}$.

6. Compute A^{10} for the following matrices A. (Hint: find eigenvalues of A first. Choose an approach to the problem depending on whether eigenvalues are real of complex numbers).

(a) $A = \begin{bmatrix} -4 & -4 \\ 10 & 8 \end{bmatrix}$

(b) $A = \begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$

7. Let

$$u_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}, \quad u_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}.$$

Determine whether $< u_1, u_2, u_3, u_4 >$ form an orthogonal basis of $\mathbf{R^4}$. Is this an orthonormal basis?

Let

$$y = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$$

Compute coordinates of y relative to the basis $< u_1, u_2, u_3, u_4 >$.

8. Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 3 & 2 & 3 \\ 3 & 3 & 1 \end{bmatrix}$. Find basis and dimension of $(Col\ A)^{\perp}$.

9. Let $A = \begin{bmatrix} -\sqrt{3}/2 & 0.5 \\ -0.5 & -\sqrt{3}/2 \end{bmatrix}$.

- Describe geometrically the linear transformation $T : \mathbf{R^2} \to \mathbf{R^2}$ defined by the matrix A.

- Describe geometrically the transformaion $T^{(10)}$ (the 10-th iteration of T).

- Write down the matrix A^{10}.