MATH 342 Lin Alg II Winter 2004
Practice problems **Name:** ____________________

1. Let a, b be positive real numbers. Using Cauchy-Schwartz inequality, prove that
$$\sqrt{ab} \le \frac{a+b}{2}$$

2. Compute the QR-factorization of $A = \begin{bmatrix} 0 & 1 & -2 \\ 3 & 2 & 3 \\ -1 & -1 & 1 \end{bmatrix}$.

3. Find least square solutions and least square errors for $Ax = b$ in three ways:
   - computing the orthogonal projection of b onto $Col A$
   - constructing the normal equations
   - using the QR-factorization of A

   (a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$.

   (b) $A = \begin{bmatrix} 4 & 0 & 1 \\ 1 & -5 & 1 \\ 6 & 1 & 0 \\ 1 & -1 & 5 \end{bmatrix}$, $b = \begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

4. Find A^{10} for $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$.

5. Make a change of variable $x = Py$ which transform quadratic forms below into quadratic forms without cross-product terms. Give P and new quardatic forms. Classify quadratic forms.

   (a) $6xy + 8yz$.

   (b) $-5x^2 + 4xy - 2y^2$.

6. Let A be a symmetric 2×2 matrix. Prove that eigenvalues of A are real.

7. Let $\mathbf{P}_3$ have the inner product given by evaluation at $(-2, -1, 1, 2)$. Find the best approximation to $t^3 + 17t$ by polynomials in the span of 1, t, $t^2 - 81t + 100$.

8. Find the maximal and minimal values of the function $f(x, y, z) = x + y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$.