MATH 342 Lin Alg II Winter 2004
Midterm **Name:** ______________________

No book, notes or calculators are alllowed. Show ALL your work.

(20) 1. Let $A = \begin{bmatrix} -0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}$

(a)[5] Find eigenvalues and eigenvectors of A.

(b)[5] Describe geometrically the linear transformation of the plane defined by the matrix A.

(c)[10] Compute A^8. Describe geometrically the linear transformation defined by A^8.

(d) (**Bonus**: 5pt.). Draw a typical trajectory of the dynamical system defined by matrix A. Is the origin an attractor or a repeller?

(20) 2. Let

$$A = \begin{bmatrix} 0.4 & 0.3 \\ -0.5 & 1.2 \end{bmatrix}$$

be the matrix of the discrete dynamical system $x_{k+1} = Ax_k$ describing relative population of spotted owls and (thousands of) flying squirrels in the old-growth forest of Douglas fir. Show that both owls and squirrels will eventually perish. What should be the initial ratio between the numbers of owls and (thousands of) squirrels so that they perish the fastest? The slowest? Classify the origin (point $(0,0)$) for this dynamical system (an attractor, a repeller or a saddle point).

You may use $ch(A) = (0.4-\lambda)(1.2-\lambda) + 0.15 = \lambda^2 - 1.6\lambda + 0.63 = (\lambda - 0.7)(\lambda - 0.9)$.

(10) 3. Find eigenvalues of the matrix

$$A = \begin{bmatrix} -1 & 0 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{bmatrix}.$$

Is this matrix diagonalizable? Explain (you do NOT have to diagonalize).

(20) 4. Let

$$A = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}.$$

(a)[10] Diagonalize A if possible.

(b)[10] Compute A^{289}.

(30) 5. Let

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

(a)[10] Determine whether $< u_1, u_2, u_3 >$ form an orthogonal basis of $\mathbf{R^3}$. Is this an orthonormal basis?

(b)[10] Let

$$y_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad y_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

Let (c_{ij}) be the coordinates of y_i relative to the basis $< u_1, u_2, u_3 >$, i.e.

$y_1 = c_{11}u_1 + c_{12}u_2 + c_{13}u_3,$

$y_2 = c_{21}u_1 + c_{22}u_2 + c_{23}u_3,$

$y_3 = c_{31}u_1 + c_{23}u_2 + c_{33}u_3.$

Find coefficients c_{ij}. Arrange your answer as a 3×3 matrix C with entires c_{ij}.

(c)[10] Let $T : \mathbf{R^3} \to \mathbf{R^3}$ be the linear transformation defined by the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Compute the matrix of T relative to the basis $< u_1, u_2, u_3 >$.

(d) (**Bonus: 5 points.**) If your arithmetic worked out correctly, the resulting matrices in (b) and (c) should be transpose to each other. Unless you already used (b) to do (c) (in which case you get your bonus points automatically), explain why this happened.