Complementary homework I

due Wednesday, April 6

(1) Compute characteristic equation, eigenvalues and eigenvectors:

(a) $A = \begin{pmatrix} 1 & -4 \\ 4 & 2 \end{pmatrix}$

(b) $A = \begin{pmatrix} 1 & 4 \\ 4 & 2 \end{pmatrix}$

(c) $A = \begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix}$

(d) $A = \begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$

(e) $A = \begin{pmatrix} 2 & -4 \\ 5 & 3 \end{pmatrix}$

(f) $A = \begin{pmatrix} 2 & -4 \\ -4 & 3 \end{pmatrix}$

(2) **Definition.** A square matrix A is called *symmetric* if $A = A^T$.

Observe that matrices in (1.f), (1.d) and (1.b) are symmetric and have real eigenvalues. Show that this will always be the case for a 2x2 matrix (in fact, this will hold for any symmetric matrix). In other words, prove that eigenvalues of a matrix A of the form $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$ are always real numbers.

(3) Let $A = \begin{pmatrix} 1 & b \\ c & 2 \end{pmatrix}$ be a matrix of rank 1. Find eigenvalues of A.

(4) Let A be a 3x3 matrix with $tr\, A = 1$, $det\, A = -1$, and integer eigenvalues. Find the eigenvalues.