MATH 256, Winter 2004
PRACTICE Midterm **Name:** ______________________

1. Find general solutions of the ODE

   (a) $y' - 2y = e^{3t}$

   (b) $y' = y^2/t$

   (c) $y' = y(y+1)/t$

   (d) $y' = y - y^3$ (Bernoulli equation)

2. Solve initial value problems. Determine the longest interval where solution is well-defined. For some of the questions, describe the behaviour of the solution when $t \to \infty$ (long-term behavior).

   (a) $y' = t, \quad y(2) = 5$

   Describe behaviour at ∞.

   (b) $y'' + e^t y' + t^3 y = 0, \quad y(1) = 0, y'(1) = 0$

   (c) $ty' + 2y = \sin t, \quad y(1) = \sin 1 - \cos 1 + 1$

   Describe behaviour at ∞.

   (d)$y' = 2t/(y + t^2 y), \quad y(0) = -2$

   (e) $y' = 2y - 5y^3, \quad y(0) = 1$ (Bernoulli equation)

3. Find a fundamental set of solutions, compute Wronskian, give a general solution and then solve the initial value problem, sketch the graph of the solution and describe behaviour at ∞.

   (a)$y'' + 5y' + 6y = 0, \quad y(0) = 2, y'(0) = 2$

   (b)$y'' + 2y' + 6y = 0, \quad y(0) = 2, y'(0) = 2$

4. Someone is buying a house anticipating to be able to pay $1200(1 + t/60)$ where t is the number of months since the loan was made. Assuming the interest rate to be 5%, compute the price the person can afford if he wishes to pay off the loan in 20 years.

5. It is being observed that the number of poorely designed ballots at the presidential elections that later have to be disqualified grows by 20% with every election (or 5% yearly if one assumes that ballots are made continuously between the elections). Assuming that the number of registered voters remains constant over the years and that the number of faulty ballots in 2000 consituted 5% of all ballots, predict in what year the elections will come to a halt because of no qualified ballots to count. You may use $\ln 20 \simeq 3$.

6. For the following equations, draw the phase line, determine equilibrium solutions, classify them as asymptotically stable, unstable or semistable. Sketch several graphs of solutions on the ty-plane. Find general solution of the equation.

(a) $y' = -17(y-2)^2$

(b) $y' = (y-1)(y-3)$

7. Approximate the solution of the initial value problem $y' = 2y - 1, \quad y(1) = 0$ at $t = 2$. Take $h = 0.25$. Solve the equation, compare your approximation with the exact value of the solution at $t = 2$ and estimate the error.

8.

Is it possible that this picture represents integral curves for the equation

(a) $y' = y^{1/3}$

(b) $y' = y^{4/3}$

(c) $t^2 y'' = y$

(d) $y'' = t^2 y$

9. State the longest interval where the solution of the initial value problem

$$(t+1)y'' + \frac{1}{t-1}y' + (\ln t)\, y = 0, \quad y(1/2) = 0, y'(1/2) = 1$$

exists. Is solution unique? Explain.

10. Does the initial value problem

$$y' = t^2 + y^2, \quad y(1) = 2, y'(1) = 239$$

have unique solution?