MATH 256, Fall 2004
Midterm **Name:** ____________________

(20) 1. Find general solutions of the ODE

(a) $y' - 2y = e^{3t}$

(b) $y' = y^2/t$

(20) 2. Solve the initial value problem. Predict long term behaviour of the solution (behaviour at ∞).

$y'' - 5y' + 6y = 0, \quad y(0) = 3, y'(0) = 7.$

3. Check all terms that apply to the equation

$$y'' - 2ty = 0$$

1. differential equation
2. first order
3. second order
4. linear
5. non-linear
6. autonomous
7. separable
8. homogeneous
9. non-homogeneous

(For each marked item you get +1pt if it applies and -1pt if it does not)

(8) 4. (a) Give an example of a second order linear non-homogeneous equation with non-constant coefficients.

(b) Does the following apply to your example above : " if $y_1(t), y_2(t)$ are solutions of the equation then $ay_1(t) + by_2(t)$ is a solution for any choice of numbers a, b"?

(20) 5. $y' = y^2 - 4$.

(a) Find equilibrium solutions

(b) Draw the phase line

(c) Classify equilibrium solutions as stable, unstable or semistable

(d) Sketch several solutions on the ty plane. Using your sketch, predict long time behaviour (behaviour at ∞) of the solution of the initial value problem $y(0) = -3/2$.

(20) 6. It is being observed that the number of poorly designed ballots at the presidential elections that later have to be disqualified grows by 20% with every election (or 5% yearly if one assumes that ballots are made continuously between the elections). Assuming that the number of registered voters, and, thus, the total number of ballots, remains constant over the years and that the number of faulty ballots in 2000 consituted 5% of all ballots, predict in what year the elections will come to a halt because of no qualified ballots to count. You may use $\ln 20 \simeq 3$.

(7) 7. Explain why the picture below cannot represent integral curves of the equation $y' + e^{t^2} y = 0$

(10) 8. (Bonus.) Consider an equation $y'' + p(t)y' + q(t) = 0$, where $p(t), q(t)$ are continuous functions. Let $(y_1(t), y_2(t))$ be a fundamental set of solutions, and let $(u_1(t), u_2(t))$ b another fundamental set of solutions. Compute the first derivative of the function $\frac{W(y_1,y_2)}{W(u_1,u_2)}$