1. Solve the initial value problem
   \[ y' = t, \quad \text{quady}(0) = 0 \]
   **Answer.** \[ y = t^2 / 2. \]

2. \[ y' + 2y = e^{-t} \]
   **Answer.** \[ y = e^{-t} + ce^{-2t} \]

3. \[ y' = 2t/(y + t^2y) \]
   **Answer.** \[ y^2 = 2 \ln(t^2 + 1) + c \]

4. \[ y^{(4)} - 8y' = 0 \]
   **Answer.** \[ c_1 + c_2 e^{2t} + c_3 e^{-t} \cos(\sqrt{3}t) + c_4 e^{-t} \sin(\sqrt{3}t) \]

5. Consider the differential equation \[ y'' + 5y' + 6y = 0. \]
   (10) (a) Find fundamental set of solutions
   **Answer.** \[ < e^{-3t}, e^{-2t} > \]
   (10) (b) Compute Wronskian
   **Answer.** \[ W = e^{-5t} \]
   (5) (c) Are the following pairs of functions linearly independent? Give a short explanation
   (c.1) \[ < e^{-3t}, e^{-2t} > \]
   **Answer.** Yes. \( W(e^{-3t}, e^{-2t}) \neq 0 \), thus, they are linearly independent.
   (c.2) \[ < -3e^t, -2e^t > \]
   **Answer.** No. We have \(-2e^t = (\frac{2}{3})(-2e^t)\). Thus, functions are linearly dependent.
6. Find the general solution of
\[ y'' - 5y' + 6y = e^{2t} \]
\[ \text{Answer. } y = c_1 e^{3t} + c_2 e^{2t} - te^{2t} \]

7. Consider the differential equation
\[ y' = 2y - y^2. \]

(1) (a) Is this an autonomous equation? \textbf{Answer.} Yes.

(1) (b) Is this a separable equation? \textbf{Answer.} Yes.

(1) (c) Is this a linear equation? \textbf{Answer.} No.

(3) (d) Find equilibrium solutions.
\[ \text{Answer. } y(t) = 0, y(t) = 2 \]

(7) (e) Draw the phase line and sketch several integral curves.

(3) (f) Classify equilibrium solutions as asymptotically stable, semistable or unstable.
\[ \text{Answer. } y(t) = 0 \text{ is unstable, } y(t) = 2 \text{ is asymptotically stable.} \]

(10) (g) Consider the initial value problem \[ y' = 2y - y^2, \ y(0) = 1 \] (still the same equation). Using Euler’s method with the step \( h = 0.5 \), approximate the solution at the point \( t = 1 \).
\[ \text{Answer. } \text{Let } f(y) = 2y - y^2. \]
\[ y_0 = 1 \]
\[ y_1 = y_0 + f(y_0)0.5 = 1.5 \]
\[ y_2 = y_1 + f(y_1)0.5 = 1.5 + (2 \cdot 1.5 - 1.5^2) = 1.5 + 0.75 \cdot 0.5 = 1.875 \]
(alternatively, in case you prefer fractions, \( y_2 = \frac{3}{2} + (2 \cdot \frac{3}{2} - (\frac{3}{2})^2) \cdot \frac{1}{2} = \frac{3}{2} + \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{2} + \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8} \))

(2) (h) Describe long term behaviour of the solution of the initial value problem above.
\[ \text{Answer. } y(t) \to 2 \]

(2) (i) For a small step \( h \), would you expect the Euler’s method to produce good approximation for this particular initial value problem? Explain.
\[ \text{Answer. } \text{Yes, because } y(t) \text{ converges to 2} \]
8. Find a general solution of the ODE:

\[ y'' + y = \frac{1}{\sin t} \]

**Answer.** \( c_1 \cos t + c_2 \sin t - t \cos t - \sin t \ln |\sin t| \)

9. Give an example of a differential equation which has solutions \( e^t, te^t \).

**Answer.** \( y'' - 2y' + y = 0 \)

10. Find the first 5 terms of the power series solution of the equation \( y'' - 2xy = 0 \) at \( x = 0 \).

**Answer.** \( y(x) = a_0 + a_1 x + a_0 \frac{x^3}{3} + a_1 \frac{x^4}{6} + \ldots \)