

No notes, books or calculators. SHOW ALL YOUR WORK

- (10) 1. Solve the initial value problem

$$y' = t, \quad y(0) = 0$$

- (20) 2. Find the general solution of

$$y' + 2y = e^{-t}$$

- (20) 3. Find the general solution of
$$y' = 2t/(y + t^2y)$$

- (20) 4. Find the general solution of
$$y^{(4)} - 8y' = 0$$

(25) 5. Consider the differential equation $y'' + 5y' + 6y = 0$.

(10) (a) Find a fundamental set of solutions

(10) (b) Compute Wronskian

(5) (c) Are the following pairs of functions linearly independent? Give a short explanation

(c.1) $\langle e^{-3t}, e^{-2t} \rangle$

(c.2) $\langle -3e^t, -2e^t \rangle$

(20) 6. Find the general solution of

$$y'' - 5y' + 6y = e^{2t}$$

- (30) 7. Consider the differential equation $y' = 2y - y^2$.
- (1) (a) Is this an autonomous equation?
 - (1) (b) Is this a separable equation?
 - (1) (c) Is this a linear equation?
- (3) (d) Find equilibrium solutions.
- (7) (e) Draw the phase line and sketch several integral curves.
- (3) (f) Classify equilibrium solutions as asymptotically stable, semistable or unstable.
- (10) (g) Consider the initial value problem $y' = 2y - y^2$, $y(0) = 1$ (still the same equation). Using Euler's method with the step $h = 0.5$, approximate the solution at the point $t = 1$.

(2) (h) Describe long term behaviour of the solution of the initial value problem above.

(2) (i) For a small step h , would you expect the Euler's method to produce good approximation for this particular initial value problem? Explain.

(20) 8. Find the general solution of the ODE:

$$y'' + y = \frac{1}{\sin t} + \sin t$$

- (15) 9. Give an example of a differential equation which has solutions e^t, te^t .

(20) 10. Find the first 5 terms of the power series solution of the equation $y'' - 2xy = 0$ at $x = 0$.

(10) 11. (Bonus.) Find a general solution of the differential equation

$$y' = y - y^3.$$