

- Find an equation of the tangent line to the given curve at the given point:
  - $x^2 + xy + y^2 = 7$ ,  $(3, -2)$ ,
  - $\frac{1}{x+1} + \frac{1}{y+1} = 1$ ,  $(1, 1)$ .
- Find linear approximation  $L(x)$  to the given function at the given point:
  - $f(x) = \cos x$ , at  $(0, 1)$ ,
  - $f(x) = (1 - x)^3$ , at  $(0, 1)$ ,
  - function  $y(x)$ , given implicitly via an equation  $x^3 + y^3 = 3xy$ , at  $(3, 3)$ .
- Estimate given number by linear approximation.
  - $\sqrt[5]{31}$ ,
  - $\cos 47^\circ$ ,
  - $80^{\frac{3}{4}}$ .
- Estimate by linear approximation the change of the area  $A = \pi r^2$  of a circle, if its radius is decreased from 10 cm to 9.8 cm.
- Sketch the graphs of the following functions. Indicate the local maxima and minima, inflection points and asymptotes. Show the concave structure.
  - $x^4 - 32x$ ,
  - $\frac{x}{x^2 - x - 2}$ ,
  - $\frac{x^2 + 1}{x^2 - 4}$ ,
  - $2x^3 - 3x^2 - 36x$ ,
  - $x\sqrt{x - 3}$ .
- Differentiate (use logarithmic differentiation when appropriate)
  - $y$  if  $y = \ln(x^2 + y^2)$
  - $\sqrt[4]{\frac{x^2 + 1}{x^2 - 1}}$
  - $\frac{\sin^2 x \tan^4 x}{(x^2 + 1)}$
  - $x^{\sin x}$
  - $(\sin x)^x$
  - $\arctan(\ln x)$
- The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm<sup>2</sup>/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm<sup>2</sup>?
- Show that the equation  $x^5 + x = 5$  has exactly one real solution.

9. Using implicit differentiation calculate  $dy/dx$  and  $d^2y/dx^2$  for the curve  $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 1$ .
10. You need to manufacture a cylindrical pot, without a top, with a volume of  $1 \text{ ft}^3$ . The cylindrical part of the pot is to be made of aluminum, the bottom of copper. Copper is five times as expensive as aluminum. What dimensions would minimize the total cost of the pot?
11. Each page of a book will contain  $30 \text{ in}^2$  of print, and each page must have 2-in. margins at top and bottom and 1-in. margins at each side. What is the minimal possible area of such a page?