

**MATH 251, Fall 2003**  
**Final Exam**

**Name:** \_\_\_\_\_

Please, write clearly. No books, notes or calculators are allowed. **SHOW ALL YOUR WORK.**

(20) 1. Compute the following limits

a)  $\lim_{x \rightarrow \infty} x^{\frac{1}{3x}}$

b)  $\lim_{x \rightarrow 0} \frac{x^2}{\sin x}$

- (20) 2. State the definition of the derivative of a function  $f(x)$  at a point  $x = a$ . Using ONLY the definition, compute the derivative of  $x^2$  at  $x = 2$ .

(40) 3. Differentiate the following functions

a)  $\frac{(x^3+1)(x-2)^5(x+1)}{(x-5)^6(x-1)^2}$

b)  $\arcsin x + \arccos x$

c)  $(\sin x)^{\cos^2 x}$

d) Compute the second derivative of  $\tan x$

(20) 4. Find an equation of the tangent line to the curve

$$\frac{1}{x} + \frac{1}{y} = 1$$

at the point  $(2, 2)$ .

- (40) 5. Sketch the graph of the function  $\frac{x}{(x+2)^2}$ . Find and classify critical points, indicate the intervals where the function is increasing/decreasing, concave up/down, indicate inflection points, behaviour at infinity, domain, and vertical and horizontal asymptotes.

(20) 6. Show that the equation  $x^5 + x = 5$  has exactly one real solution.

(20) 7. Estimate  $\sin(\frac{\pi}{314})$  by linear approximation.



- (20) 8. The bottom of a closed rectangular box is twice as long as it is wide. The volume of the box is  $576\text{cm}^3$ . Find the dimensions of the box which would minimize its total surface area.