Sample MIDTERM 2
Last year’s midterm for Spring MATH 126 A, B

Scientific, but not graphing calculators are OK.
You may use one 8.5 by 11 sheet of handwritten notes.

Problem 1. Consider a particle traveling according to the equations

\[ x(t) = \cos^2 t, \quad y(t) = \cos t. \]

Write down and simplify (but do not evaluate) the formula for the length of the curve along which the particle is moving.
Problem 2. Consider a particle whose velocity, at time \( t \geq 0 \), is given by
\[
\vec{v}(t) = \langle -2t, -\sin t \rangle
\]
and whose position at \( t = 0 \) is \( (4, 0) \).

a. Find the formula for the position of the particle at time \( t \).

b. Find the point at which the particle crosses the \( y \)-axis.

c. Suppose the acceleration suddenly drops to 0 at the time when the particle crosses the \( y \)-axis, so that there are no forces acting on the particle. Find the position of the particle one minute later.
Problem 3. Find the equations of the normal and of the osculating planes to the curve

\[ \mathbf{r}(t) = (t^3, \sin(\pi t), t + 1) \]

at the point corresponding to \( t = 2 \).
Problem 4. Identify the curve 

\[ r = 2 \sin \theta + 2 \cos \theta \]

by finding a Cartesian equation for the curve. Give a verbal description of what that curve is.
**Problem 5.** Consider the function of two variables

\[ f(x, y) = \sqrt{1 + x - y^2}. \]

a. Identify and sketch the domain of \( f(x, y) \).

b. Find the partial derivatives \( f_y(x, y) \) and \( f_x(x, y) \).

c. Find the second partial derivative \( f_{xy}(x, y) \).

d. Find an equation of the tangent plane at the point \( (1, 1) \).