

MIDTERM II
Math 126, Section C
November 17, 2006

Problem	Total Points	Score
1	9	
2	15	
3	12	
4	12	
5	12	
Total	60	
6(Bonus)	2	

- You may use a scientific calculator and one two-sided sheet of handwritten notes. No other notes, books or calculators are allowed. Please turn off your cell phone.
- Show all your work to get full credit.
- Read instructions for each problem CAREFULLY.
- Leave all your answers in EXACT form.
- Check your work!

1. (9pts) A curve is given by the equation

$$r + \frac{1}{r} = 4 \cos \theta$$

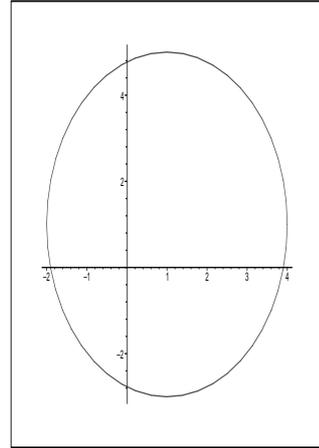
in polar coordinates.

(a) (7pts) Find the equation of the curve in Cartesian coordinates. Sketch the curve.

(b) (2pts) Using your sketch, find all the points on the curve where the tangent line is horizontal.

2. (15pts) Consider an ellipse given by the equation

$$\frac{(x - 1)^2}{9} + \frac{(y - 1)^2}{16} = 1$$



(a) (5pts) Find parametric equations of the ellipse.

(b) (5pts) Set up, but do not evaluate, the integral computing the length of the arc of the ellipse going clockwise from the point (1, 5) to the point (4, 1).

- (c) (5pts) Using the sketch of the ellipse, find the points where the curvature of the ellipse is minimal. Compute the value of the curvature at these points.

3. (12pts) A particle is moving with the acceleration $\vec{a}(t) = (-\cos t, -\sin t, 0)$, $t \geq 0$. The initial position of the particle is $(1, 0, 0)$ and the initial velocity is $(0, 1, 1)$

(a) (6pts) Find the position vector $\vec{r}(t)$.

(b) (6pts) Find the length of the projection of the acceleration vector onto the unit tangent vector.

4. (12pts) Let $f(x, y) = \sqrt{11 - x^2 - y^2}$.

(a) (3pts) Sketch a contour map for the function f showing three level curves at the level $k = 0, 1, 2, 3$. Label the curves with the corresponding values of f .

(b) (3pts) Compute $f_x(x, y)$.

(c) (3pts) Compute $f_y(x, y)$.

(d) (3pts) Find an equation of the tangent plane to the graph of f at the point $(1, 1, 3)$.

5. (12pts) Consider a curve given by the parametric equations $(t, t^2 - 1, t^3 + 1)$.

(a) (8pts) Find an equation of the normal plane at the point $(0, -1, 1)$.

(b) (4pts) Find the angle between the normal and osculating planes at the point $(1, 0, 2)$.

6(Bonus) (2pts) Let $\vec{r}(t) = (\cos t \sin t, \sin^2 t, \cos t)$. Show that \vec{r}' is perpendicular to \vec{r} at any point on the curve WITHOUT computing \vec{r}' .