Sample MIDTERM II, version 1
Last year's midterm for Spring MATH 126 A, B

Scientific, but not graphing calculators are OK.

You may use one 8.5 by 11 sheet of handwritten notes.

Problem 1. Consider a particle traveling according to the equations

$$x(t) = \cos^2 t, \quad y(t) = \cos t.$$

Write down and simplify (but do not evaluate) the formula for the length of the curve along which the particle is moving.

Problem 2. Consider a particle whose velocity, at time $t \geq 0$, is given by

$$\vec{v}(t) = \langle -2t, -\sin t \rangle$$

and whose position at $t = 0$ is $(4, 0)$.

a. Find the formula for the position of the particle at time t.

b. Find the point at which the particle crosses the y axis.

c. Suppose the acceleration suddenly drops to 0 at the time when the particle crosses the y-axis, so that there are no forces acting on the particle. Find the position of the particle one minute later.

Problem 3. Find the equations of the <u>normal</u> and of the <u>osculating</u> planes to the curve

$$\vec{r}(t) = \langle\, t^3 \,,\, \sin(\pi t) \,,\, t + 1 \,\rangle$$

at the point corresponding to $t = 2$.

Problem 4. Identify the curve
$$r = 2\sin\theta + 2\cos\theta$$
by finding a Cartesian equation for the curve. Give a verbal description of what that curve is.

Problem 5. Consider the function of two variables

$$f(x, y) = \sqrt{1 + x - y^2}.$$

a. Identify and sketch the domain of $f(x, y)$.

b. Find the partial derivatives $f_y(x, y)$ and $f_x(x, y)$.

c. Find the second partial derivative $f_{xy}(x, y)$.

d. Find an equation of the tangent plane at the point $(1, 1)$.