Quiz 6 Solutions

(a) Find the linear approximation of the function $f(x, y) = e^{5-3x^2-y^2}$ at the point $(1, 1)$ and use it to estimate $f(1.1, 0.8)$

Solution: The linear approximation is given by

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

In this case $a = 1, b = 1$ and so $f(1, 1) = e^{5-3-1} = e$. Also

$$f_x(x, y) = -6xe^{5-3x^2-y^2}, \text{ and}$$

$$f_y(x, y) = -2ye^{5-3x^2-y^2}.$$

Consequently,

$$f_x(1, 1) = -6e, f_y(1, 1) = -2e.$$

Therefore

$$f(x, y) \approx e - 6e(x - 1) - 2e(y - 1).$$

Since $(1.1, 0.8)$ is close to $(1, 1)$ we have

$$f(x, y) \approx e - 6e\frac{1}{10} - 2e\frac{-2}{10} = \frac{4}{5}e.$$

(b) Use the same linear approximation to estimate the value of x such that $f(x, 1.06) = e$

Solution: Using the approximation from (a) we have

$$e \approx e - 6e(x - 1) - 2e\frac{6}{100}.$$

Solving for x we obtain

$$x \approx \frac{49}{50} = 0.98.$$