Solutions to Math 126A Quiz 5

1. *Find the Cartesian equation of the curve given by the following equation in polar coordinates:*

$$r = 2\sin\theta$$

Sketch the curve.

Solution. Using the conversion formula $y = r\sin\theta$, the equation becomes

$$r = 2(\frac{y}{r}).$$

That implies,

$$r^2 = 2y.$$

Now using the conversion formula $r^2 = x^2 + y^2$, we get the Cartesian equation of the curve,

$$x^2 + y^2 = 2y.$$

Further by completing the square we get

$$\mathbf{x^2 + (y-1)^2 = 1.}$$

We can see that the curve is a circle centered at $(0, 1)$ of radius 1.

2. *Consider the curve given by the vector equation* $\mathrm{r}(t) = (\cos t, \sin t, 2t)$.

(a) *Find the length of the arc of the curve between the $t = 0$ and $t = \pi$ (one half of a revolution).*

Solution. The arclength of the curve is given by $s = \int_0^\pi |r'(t)|\ dt$.
Now $r'(t) = (-\sin t, \cos t, 2)$, which implies, $|r'(t)| = \sqrt{(\sin t)^2 + (\cos t)^2 + (2)^2} = \sqrt{5}$.
Thus arclength $s = \int_0^\pi \sqrt{5}\ dt = \mathbf{\sqrt{5}}\pi$.

(b) *Is parameterization $r(t) = (\cos t, \sin t, 2t)$ a natural parameterization of the curve?*
Justify your answer.

Answer. NO, the given parameterization of the curve is not a natural parameterization, since $|r'(t)| \neq 1$, as shown in part (a).

Remark. A natural parametrization is a parameterization with respect to the arclength. Thus a parameterization is natural if $|r'(t)| = 1 \iff s(t) = \int_0^t |r'(v)|\ dv = t$.