1. Consider a parametric curve on the plane given by the equations \((x(t), y(t)) = (17 + t^2, t^2 + t^3)\).

(a) Find \(\frac{dy}{dx}\), \(\frac{d^2y}{dx^2}\) as functions of \(t\).

**Solution:** First we compute

\[
\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 2t + 3t^2.
\]

Now apply the formulas for the first and second derivatives of parametric equations:

\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3t^2}{2t} = 1 + \frac{3}{2}t.
\]

\[
\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left( 1 + \frac{3}{2}t \right)}{2t} = \frac{3}{4t}.
\]

(b) Find all values of \(t\) for which the curve is concave upward.

**Solution:** Recall from Math 124 that the graph of a function is concave up on an interval \(I\) whenever \(\frac{d^2y}{dx^2} \geq 0\) on \(I\). From (a), this means the curve is concave up whenever

\[
\frac{3}{4t} \geq 0.
\]

But this occurs as long as \(t > 0\).

(c) Set up, but do not evaluate, the integral to compute the length of the arc of the curve between \(t = 0\), and \(t = 1\).

**Solution:**

\[
L = \int_0^1 \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \, dt
\]

\[
= \int_0^1 \sqrt{(2t)^2 + (2t + 3t^2)^2} \, dt
\]

\[
= \int_0^1 \sqrt{4t^2 + 4t^2 + 12t^4 + 9t^4} \, dt
\]

\[
= \int_0^1 \sqrt{8t^2 + 12t^3 + 9t^4} \, dt.
\]