Name ______________________ Section ________

Practice problems for Midterm I

Math 126, Section A

January, 2007

For the actual midterm you may use one 8×11.5 sheet of *handwritten* notes. You may use your "simple" scientific calculator on the exam. No books, printed notes or graphing calculators. You have to show ALL YOUR WORK to get full credit.

The Midterm I will cover material in the Taylor Notes and Sections 12.1-12.4 in the Stewart book. Before taking the exam, you should be able to complete and understand all homework problems, all examples from class, quiz 2 (solutions are posted on the class webpage), the problems from this list, and any problem from one of the old midterms which can be found at http://www.math.washington.edu/~m126/midterms/midterm1.php

There is no Quiz on Tuesday, January 20th: your TA will conduct a review session for the Midterm.

Practice Problems.

1. (a) Find the quadratic approximation for the function $f(x) = \ln(x)$ based at e.
   (b) Use the polynomial from (a) to estimate $\ln(3)$. Compute the error bound for your approximation. Give your answers in both exact and decimal forms.
2. Let $f(x) = e^x$, and let $T_n(x)$ be the n^{th} Taylor polynomial for $f(x)$ based at $a = 0$. Find n such that the error $|T_n(x) - e^x| \leq 0.001$ on the interval $[-1, 1]$.
3. Find Taylor series and the interval of convergence for the following functions
   (a) $f(x) = \frac{1}{(1-x)^3}$ at $b = 0$
   (b) $f(x) = \frac{x^2-3x-4}{(2x-3)(x^2+4)}$ at $b = 0$
   (c) $f(x) = e^{3x-2}$ at $b = 2$
   (d) $f(x) = 1 - 6x + 2x^{17} - x^{90}$ at $a = 0$.
   (e) $f(x) = \cos^2(x)$ at $a = 0$

(f) $f(x) = \cos(x^2)$ at $a = 0$

(g) $f(x) = xe^x$ at $a = 2$

(h) $f(x) = \int\limits_0^x \frac{e^t - 1}{t} dt$

4. Let $f(x) = x^2 \ln(1 + x^3)$.

   (a) Find the Taylor series for $f(x)$ based at $a = 0$.

   (b) Explicitly compute the coefficient by x^{17} in the Taylor expansion from (a).

   (c) Find $f^{17}(0)$.

5. Find the fourth Taylor polynomial based at $b = 0$ of the function $f(x) = \frac{e^{x^2}}{x^2 - 1}$.

6. Approximate the integral $\int\limits_0^2 \sin(x^2) dx$ using the first three non-zero terms of the Taylor series.

7. (Bonus) Approximate the integral $\int\limits_0^2 \sin(x^2) dx$ with the accuracy 10^{-4}.

8. Let $\bar{u} = (3, 2, -1)$, $\bar{v} = (2, -2, 2)$. Find a unit vector $\bar{w}$ perpendicular to both $\bar{u}$ and $\bar{v}$.

9. (a) Check if the points $A = (5, 1, 3)$, $B = (7, 9, -1)$, $C = (1, -15, 11)$ are colinear in TWO different ways.

   (b) Now let $A = (1, 2, -3)$, $B = (3, 4, -2)$, $C = (3, -2, 1)$. Check whether the triangle ABC has an obtuse angle. Find the area of the triangle.

10. (a) Show that the equation
$$x^2 + y^2 + z^2 = 4x + z$$
represents a sphere, and find its center and radius.

    (b) Check that the point $A = (4, 0, 1)$ is on the sphere from (a). Then show that the point $B = (3, 2, 5)$ belongs to the plane tangent to the sphere at the point A.
    Hint: Tangent plane to the sphere at the point A is perpendicular to the radius connecting the center to the point A.

11. (Bonus). Express x^{17} as a sum of powers of $(x - 1)$.