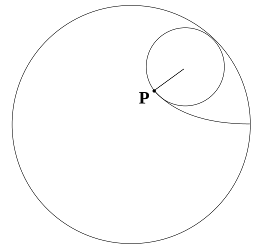
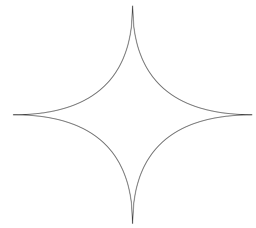
Solids of Revolution and the Astroid Math 125

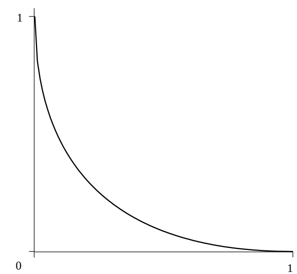
Name ________________________ Quiz Section ______





In this worksheet we are going to practice computing some more volumes of solids of revolution. These will all be based on a curve called the "astroid". This curve is formed by rolling a small wheel around the inside of a larger one (see the picture). If the radius of the small wheel is one quarter the radius of the big one, a point P on the small wheel will trace out the four pointed curve shown on the far right. It's called the astroid because it looks like a star.

1 If the radius of the big wheel is taken to be one, the astroid can be shown to have the equation $x^{2/3} + y^{2/3} = 1$. Use disks to compute the volume of the solid generated by rotating the part of the astroid in the first quadrant around the y-axis.



2 Use cylindrical shells to compute the volume of the solid generated by rotating the first quadrant portion of the astroid about the x-axis. (Hint: Try the substitution $u^3 = y^2$, so $3u^2\,du = 2y\,dy$.) How does this compare with your answer in Problem 1? Can you explain this geometrically?

3 Use any method you wish to compute the volumes of the solids generated by rotating the first quadrant portion of the astroid about the lines $x = 1$ and $y = -1$. **Set up only. Do not compute the integrals.**

6 Archimedes (ca. 287-212 B.C.) was able to use clever geometric means to determine the relative volumes of a cylinder and the cone and paraboloid that would fit snugly into it (1800 years before Newton and Leibniz). With calculus, you don't have to be a genius to reach the same conclusions.

a) Determine the ratio of the volume of a (right circular) cone to the volume of a cylinder with the same height and base radius (Figure 1). (First you will need to determine the equation of the line which rotates to generate the cone.)

$$\frac{V_{cone}}{V_{cyl}} = \underline{\qquad\qquad}$$

b) Determine the ratio of the volume of a paraboloid (a parabola rotated about the y=axis) to the volume of a cylinder with the same height and base radius (Figure 2).

$$\frac{V_{para}}{V_{cyl}} = \underline{\qquad\qquad}$$

c) Archimedes was particularly pleased when he determined the ratio of the volumes of a sphere and cylinder. Determine the ratio of the volume of a sphere to the volume of a cylinder having the same radius, and having height the same as the diameter of the sphere (Figure 3).

$$\frac{V_{sph}}{V_{cyl}} = \underline{\qquad\qquad}$$

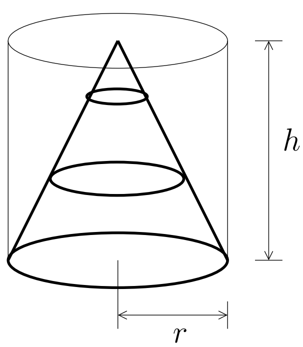


Figure 5: Cone

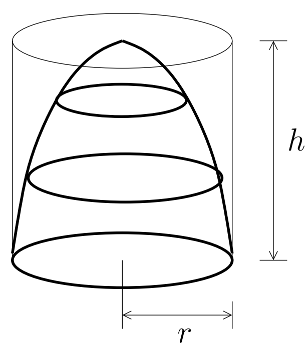


Figure 6: Paraboloid

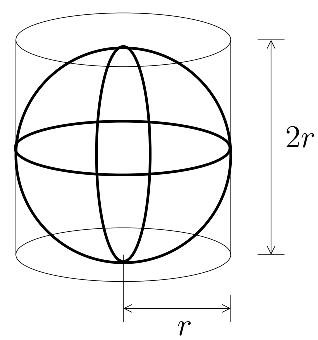


Figure 7: Sphere

7 I wanted to determine how much water my sprinkler was using, so I set a bunch of empty cat food cans out at various distances from the sprinkler and noted how much water was in each can after an hour. The data are given in the table below. The sprinkler distributes water in a circular pattern, so I assumed that points the same distance from the sprinkler received the same amounts of water.

a) Use the data in Table 1 to estimate how much water my lawn got from this sprinkler in one hour. Describe the method you are using and calculate the amount of water.

b) My neighbor decided to collect the same data for her watering, but she forgot to set out the cans at evenly spaced distances and so the calculation is a bit more complicated. Use the data in Table 2 to estimate how much water her lawn got in one hour. Describe the method you are using and calculate the amount of water.

Distance (feet)	Water (inches)
2	2.1
4	1.8
6	1.9
8	1.6
10	1.2
12	1.4
14	1.6
16	1.0

Table 1: My Lawn

Distance (feet)	Water (inches)
2	2.2
5	1.8
9	2.1
11	1.4
15	1.2
17	1.0
21	0.5

Table 2: My Neighbor's Lawn