MATH **125** Quiz **Nov. 27** **Name:**

No books, notes or graphing calculators. Turn off your cell phones. Good luck!

(5) 1. Determine whether the following integral is convergent or divergent, and evaluate it if it is convergent.

$$\int_0^3 \frac{1}{\sqrt[3]{t}}\,\mathrm{dt}$$

Solution. We first express the integral of terms of a limit:

$$\lim_{s\to 0^+}\int_2^3 \frac{1}{t^{1/3}}\mathrm{dt} = \lim_{s\to 0^+}\int_s^3 t^{-1/3} = \lim_{s\to 0^+}\left.\frac{3t^{2/3}}{2}\right|_s^3 = \lim_{s\to 0^+}\left(\frac{3\cdot 3^{2/3}}{2} - \frac{3s^{2/3}}{2}\right) = \frac{3^{5/3}}{2} - 0 = \frac{3^{5/3}}{2}.$$

Since the limit exists, the integral is convergent.

(5) 2. Find the coordinates of the center of mass of the uniform flat plate bounded by the x-axis, the graph of the function $y = \ln x$ and the vertical line $x = e$.

Solution. Step I. Find the area. We compute $A = \int_1^e \ln x\mathrm{dx} = \mathrm{x}\ln\mathrm{x} - \mathrm{x}\Big|_1^e = 1.$

Step II. Find the moment $M_y = \int_1^e x\ln x\,\mathrm{dx} = \frac{\mathrm{x}^2}{2}\ln\mathrm{x} - \frac{\mathrm{x}^2}{4}\Big|_1^e = \frac{e^2+1}{4}$

Hence, $\bar{x} = \frac{M_y}{A} = \frac{e^2+1}{4}$

Step 3. $M_x = \frac{1}{2}\int_1^e \ln^2 x\,\mathrm{dx} = \frac{1}{2}\left(\mathrm{x}\ln^2\mathrm{x} - 2\mathrm{x}\ln\mathrm{x} + 2\mathrm{x}\Big|_1^e\right) = \frac{e-2}{2}$

Thus, $\bar{y} = \frac{M_x}{A} = \frac{e-2}{2}$. Therefore, the center of mass is $(\bar{x}, \bar{y}) = \left(\frac{e^2+1}{4}, \frac{e-2}{2}\right)$.