1.)

$$\int \frac{dt}{\sqrt{t^2 - 6t + 13}}$$

$$= \int \frac{dt}{\sqrt{(t-3)^2 + 4}}$$

Let $t - 3 = 2\tan x$

The integral becomes:

$$\int \frac{2\sec^2 x dx}{\sqrt{4\tan^2 x + 4}}$$

$$= \int \sec x dx$$

$$= \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{4+(t-3)^2} + t - 3}{\sqrt{4+(t-3)^2} - t + 3}\right| + C$$

2.) Method 1 (partial fractions):

$$\int \frac{dt}{t^2 - 6t + 8}$$

$$= \int \frac{dt}{(t-2)(t-4)}$$

$$= \int \left(\frac{-1/2}{t-2} + \frac{1/2}{t-4}\right) dt$$

$$= -\frac{1}{2}\ln|t-2| + \frac{1}{2}\ln|t-4| + C$$

$$= \frac{1}{2}\ln\left|\frac{t-4}{t-2}\right| + C$$

Method 2 (trig substitution)

$$\int \frac{dt}{t^2 - 6t + 8}$$
$$= \int \frac{dt}{(t-3)^2 - 1}$$

Let $t - 3 = \sin x$, then the integral becomes:

$$\int \frac{\cos x}{\sin^2 x - 1} dx$$
$$= \int \frac{\cos x}{-\cos^2 x} dx$$
$$= -\int \frac{1}{\cos x} dx$$
$$= -\int \sec x dx$$

Use formula given in question (1):

$$-\int \sec x dx$$
$$= -\frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| + C$$
$$= -\frac{1}{2} \ln \left| \frac{1 + (t-3)}{1 - (t-3)} \right| + C$$
$$= \frac{1}{2} \ln \left| \frac{t-4}{t-2} \right| + C$$

Bonus problem:

$$\int \frac{dx}{1+\cos x}$$
$$= \int \frac{dx}{1+(2\cos^2 \frac{x}{2} - 1)}$$
$$= \frac{1}{2}\int \sec^2 \frac{x}{2}\, dx$$

Let

$$u = \tan \frac{x}{2}$$
$$\Rightarrow du = \frac{1}{2}\sec^2 \frac{x}{2}\, dx$$

Hence the integral becomes:

$$\int du$$
$$= u + C$$
$$= \tan \frac{x}{2} + C$$