MATH **125** Quiz **Oct. 23** **Name:**

No books, notes or graphing calculators. Turn off your cell phones. Good luck!

1. Consider the region in the first quadrant bounded by the curves $y = 2(x-1)^2$, $y = x^2 - 2x + 5$ and the y-axis.

(2 pt) a. Sketch the region.

Solution: See Attachment.

(8 pt) b. Find the volume of the solid of revolution obtained by rotating the region around the y-axis. State clearly which method you use: shells or washers. Sketch a typical shell or washer.

For the Shell Method:

$V = 2\pi \int_a^b xh(x)\mathrm{d}x$, and so to find the endpoints we set

$$2(x-1)^2 = x^2 - 2x + 5 \Rightarrow 2x^2 - 4x + 2 = x^2 - 2x + 5 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, x = -1$$

Thus, since the region is also bound by $y = 0$, the endpoints are $a = 0$ and $b = 3$, so we have:

$V = 2\pi \int_0^3 xh(x)\mathrm{d}x$ where $h(x)$ is the height of each slice, so

$$\begin{aligned}
V &= 2\pi \int_0^3 x(x^2 - 2x + 5 - 2(x-1)^2)\mathrm{d}x \\
&= 2\pi \int_0^3 x(x^2 - 2x + 5 - 2x^2 + 4x - 2)\mathrm{d}x \\
&= 2\pi \int_0^3 (-x^3 + 2x^2 + 3x)\mathrm{d}x = 2\pi \left(\frac{-x^4}{4} + \frac{2x^3}{3} + \frac{3x^2}{2} \Big|_0^3 \right) \\
&= 2\pi (\frac{-81}{4} + \frac{54}{3} + \frac{27}{2}) = 2\pi (\frac{126 - 81}{4}) = \frac{45\pi}{2}
\end{aligned}$$