MATH **125** Quiz **Oct. 9** **Name:**

No books, notes or graphing calculators. Turn off your cell phones. Good luck!

(5) 1. Compute the derivatives of the following functions

$$(a)\,[2\text{pt}]\quad g(x) = \int_0^x \sin t\,\mathrm{d}t, \qquad (b)\,[3\text{pt}]\quad h(x) = \int_0^{x^2} \sin t\,\mathrm{d}t.$$

Solution. 1a). By the FTC I, $g'(x) = \sin x$.

1b). Let $u(x) = x^2$, $h(u) = \int_0^u \sin t\,\mathrm{d}t$. Then $h(x) = h(u(x))$. Applying chain rule to $h(u(x))$, we get

$$h'(x) = h'(u(x))u'(x). \qquad (*)$$

By the FTC I, $h'(u) = \sin u$; we also compute $u'(x) = 2x$. Now, plug this into the formula (*) for $h'(x)$:

$$h'(x) = \sin u(x) \bullet 2x = 2x \sin x^2.$$

Since one can evaluate the integrals in 1a,b, there is an

Alternative Solution. 1a). We compute $g(x) = \int_0^x \sin t\,\mathrm{d}t = -\cos t\Big|_0^x = -\cos x + \cos 0 = 1 - \cos x$.

Now just differentiate: $\frac{\mathrm{d}}{\mathrm{dx}} g(x) = (1 - \cos x)' = \sin x$.

1b). Again, evaluate the integral: $h(x) = \int_0^{x^2} \sin t\,\mathrm{d}t = -\cos t\Big|_0^{x^2} = -\cos(x^2) + \cos 0 = 1 - \cos(x^2)$.

Now differentiate: $\frac{\mathrm{d}}{\mathrm{dx}} h(x) = (1 - \cos(x^2))' = 2x \sin(x^2)$.

(5) 2. Compute the definite integral $\int_0^{\frac{\pi}{4}} (2\sec^2\theta - 3\cos\theta)\,\mathrm{d}\theta$. Leave your answer in exact form.

Solution. $\int_0^{\frac{\pi}{4}} (2\sec^2\theta - 3\cos\theta)\,\mathrm{d}\theta = \int_0^{\frac{\pi}{4}} 2\sec^2\theta\,\mathrm{d}\theta - \int_0^{\frac{\pi}{4}} 3\cos\theta\,\mathrm{d}\theta = 2\int_0^{\frac{\pi}{4}} \sec^2\theta\,\mathrm{d}\theta - 3\int_0^{\frac{\pi}{4}} \cos\theta\,\mathrm{d}\theta = 2\tan\theta\Big|_0^{\frac{\pi}{4}} - 3\sin\theta\Big|_0^{\frac{\pi}{4}} =$
$2\tan\frac{\pi}{4} - 2\tan 0 - 3\sin\frac{\pi}{4} + 3\sin 0 = 2 - 0 - 3\frac{\sqrt{2}}{2} + 0 = 2 - 3\frac{\sqrt{2}}{2}$.

3. [Bonus problem: 1 bonus point] Compute the definite integral $\int_{-4}^{2} \sqrt{8 - 2x - x^2}\,\mathrm{d}x$

Try to do this problem when you practice for the midterm. Ask your professor or TA for help if you need it!