MATH **125** Quiz **Oct. 2** **Name:**

No books, notes or graphing calcuators. Turn off your cell phones.

(5) 1. Find the most general anti-derivative of the function

$$f(x) = \frac{1-x^2}{x}$$

SOLUTION: Note that

$$\frac{1-x^2}{x} = \frac{1}{x} - \frac{x^2}{x} = \frac{1}{x} - x,$$

so since the anti-derivative of $1/x$ is $\ln(x)$ and the anti-derivative of x is $x^2/2$, the most general anti-derivative of $f(x)$ is

$$\ln(x) - \frac{x^2}{2} + C,$$

where C is any constant.

(5) 2. Considetr the graph of the function $y = x^3$ on the interval $[0, 1]$. Estimate the area under the graph using the Right end-point Riemann sum for $n = 4$. You can use a simple calculator for this problem. Do you get an under-estimate or an over-estimate?

First break the interval $[0, 1]$ up into 4 subintervals of equal size (i.e. $[0, 1/4]$, $[1/4, 1/2]$, $[1/2, 3/4]$, $[3/4, 1]$). Then compute the Right end-point Riemann Sum as the text describes:

$$\begin{aligned} R_4 &= f\left(\frac{1}{4}\right) \times \frac{1}{4} + f\left(\frac{2}{4}\right) \times \frac{1}{4} + f\left(\frac{3}{4}\right) \times \frac{1}{4} + f\left(\frac{4}{4}\right) \times \frac{1}{4} \\ &= \frac{1}{4} \times \left(\left(\frac{1}{4}\right)^3 + \left(\frac{2}{4}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{4}{4}\right)^3\right) \\ &= \left(\frac{1}{4}\right)^4 \times (1^3 + 2^3 + 3^3 + 4^3) \\ &= \frac{100}{256} = 0.390625. \end{aligned}$$

Since x^3 is increasing on the interval $[0, 1]$, the Right end-point Riemann sum is an over-estimate for the area.

3. [Bonus problem: 1 bonus point] Find an anti-derivative of the function

$$f(x) = (1 + \ln x)x^x$$

Ask a TA or the professor during office hours for the solution.