Practice problems for Midterm I

Math 125, Sections C&D

October, 2007

For the actual midterm you may use one 8×11.5 sheet of *handwritten* notes (one-side only). You may use your "simple" scientific calculator on the exam. No books, printed notes or graphing calculators. You have to show ALL YOUR WORK to get full credit.

This is intended for practice and is much longer than the actual midterm. There are sample midterms linked to our class webpage; you can find even more old midterms at the unified MATH 125 website.

Practice Problems.

1. Find f if $f'(x) = 1 - 3\sqrt{x} + e^{-x}$ and $f(1) = \frac{1}{e} - 1$.

2. Evaluate $\displaystyle\int_0^2 (1 + \sqrt{4 - x^2})\ dx$ by interpreting it in terms of areas.

3. If $\displaystyle\int_{-1}^{7} h(x)\ dx = 10$ and $\displaystyle\int_{-1}^{3} h(x)\ dx = 5$, find $\displaystyle\int_3^7 4h(x)\ dx$.

4. Compute

   (a) $\displaystyle\int_0^2 \frac{\mathrm{d}}{\mathrm{dx}}(2\cos x^4)\ dx$

   (b) $\frac{\mathrm{d}}{\mathrm{dx}} \displaystyle\int_0^2 2\cos t^4\ dt$

   (c) $\frac{\mathrm{d}}{\mathrm{dx}} \displaystyle\int_0^{2x} 2\cos t^4\ dt$

5. Evaluate the following integrals

(a) $\displaystyle\int_0^{\frac{1}{2}} \frac{5}{\sqrt{1-x^2}}\, dx$

(b) $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

(c) $\displaystyle\int \frac{x^5}{2+\frac{1}{3}x^6}\, dx$

(d) $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1+x^3}}\, dx$

(e) $\displaystyle\int (x^3-2)\cdot \sin(x^4-8x+3)\, dx$

(f) $\displaystyle\int_{-3}^{-1} \frac{1}{5+x^2+4x}\, dx$

(g) $\displaystyle\int_{-1}^{1} \frac{2\sin^3 x}{3+2x^2+5x^8}\, dx$

6. Use the Midpoint Rule with $n = 3$ to approximate $\int_0^{\pi} \sin^2 x \, dx$. (Leave π in your answer.)

7. Let $g(x) = \int_0^{\sin x} \sqrt[3]{1 - t^2} \, dt$. Compute $g'(x)$, $g'(\pi)$ and $g(\pi)$.

8. A particle moves along a line with velocity function $v(t) = t^2 - 4t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 6]$ seconds.

9. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = \cos\left(\frac{\pi x}{2}\right)$ and $y = 1 - x$.

   (a) Sketch $\mathcal{R}$. (**Hint:** What is $\cos 0$? $\cos \frac{\pi}{2}$?)

   (b) Find the area of $\mathcal{R}$.

   (c) Set up (but **DO NOT EVALUATE**) an integral expression for the volume of the solid generated when $\mathcal{R}$ is rotated about the x-axis.

   (d) Set up (but **DO NOT EVALUATE**) an integral expression for the volume of the solid generated when $\mathcal{R}$ is rotated about the line $y = 1$.

10. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.

    (a) Sketch $\mathcal{R}$.

    (b) Compute the area $\mathcal{R}$.

    (c) Compute the volume obtained by rotating $\mathcal{R}$ around the x-axis.