Your Name

Your Signature

TA's Name #

Quiz Section

**1.** [**8 points total**] Evaluate the following integrals.

(a) [**4 points**] $\displaystyle\int \frac{x^2}{\sqrt[3]{x^3+2}}\,dx$

**Answer.** $\displaystyle\int \frac{x^2}{\sqrt[3]{x^3+2}}\,dx \overset{u=x^3+2}{=} \frac{1}{3}\int u^{-\frac{1}{3}}\,du = \frac{1}{2}u^{\frac{2}{3}} + C = \boxed{\tfrac{1}{2}(x^3+2)^{\frac{2}{3}} + C}$

(b) [**4 points**] $\displaystyle\int \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta$

**Answer.** $\displaystyle\int \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta\,dx \overset{u=\tan\theta}{=} \int \frac{1}{1+u^2}\,du = \arctan(u) + C =$
$\arctan(\tan\theta)) + C = \boxed{\theta + C}$

Alternatively, one can observe that the trigonometric identity $\sec^2 x = 1 + \tan^2 x$

implies that $\frac{\sec^2\theta}{1+\tan^2\theta} = 1$. Hence, $\displaystyle\int \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta = \int 1\,d\theta = \theta + C$

**2.** [**12 points total**] Evaluate the following integrals. Simplify as mush as possible but leave your answers in **exact form**. Do not give a decimal answer.

(a) [**4 points**] $\displaystyle\int_0^1 \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

**Note.** This question the way it is stated DOES NOT make sense. The integrand is not defined on the interval $[1,e]$ (try plugging $e$ into $\sqrt{1-u^2}$). Nonetheless we were giving full credit to people who got $\arcsin e - \pi/2$ even though arcsin is undefined at $e$. The problem was supposed to ask the following:

Evaluate $\displaystyle\int_0^1 \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$

**Answer.** $\displaystyle\int_0^1 \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx \overset{u=e^{-x}}{=} -\int_1^{1/e} \frac{1}{\sqrt{1-u^2}}\,du = \int_{1/e}^1 \frac{1}{\sqrt{1-u^2}} = \arcsin u\Big|_{\frac{1}{e}}^{1} =$
$\boxed{\frac{\pi}{2} - \arcsin\frac{1}{e}}$

(b) [**4 points**] $\int_1^e \frac{\ln x}{x}\, dx$

**Answer.** $\int_1^e \frac{\ln x}{x}\, dx \overset{u=\ln x}{=} \int_0^1 u\, du = \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$

(c) [**4 points**] $\int_{-2}^2 x(1+x^2)^{17}\, dx$

**Answer.** $\int_{-2}^2 x(1+x^2)^{17}\, dx = \boxed{0}$ because the integrand is an odd function.

**3.** [**10 points total**] Let $R$ be a region in the first quadrant bounded by the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

(a quarter of an ellipse). Use the *left end-point* Riemann sum with $n = 3$ to estimate the area of the region $R$. Draw the picture of the region and clearly sketch the three rectangles you use for approximation.

**Answer.** The $x$-intercept in the first quadrant is $x = 2$.
$\Delta x = 2/3$. The left-end points are 0, 2/3 and 4/3. The height at these points is 3, $2\sqrt{2}$ and $\sqrt{5}$ respectively. Hence, $R_3 = \boxed{\frac{2}{3}(3 + 2\sqrt{2} + \sqrt{5})} \sim 5.376$.

**4.** [**10 points total**]

A small electric car travels along a straight track. The velocity of the car is given by the function

$$v(t) = 12t - 3t^2 \text{ ft/sec}.$$

(a) [**4 points**] How far away is the car from its starting point after 5 seconds?

**Answer.** $\int_0^5 v(t)\, dt = \int_0^5 12t - 3t^2\, dt = (6t^2 - t^3)\Big|_0^5 = \boxed{25}$ m.

(b) [**6 points**] Find the *total distance* traveled by the car during the first 5 seconds.

**Answer.** First, we solve the inequality $v(t) = 12t - 3t^2 > 0$. We get that $v(t) > 0$ when $t < 4$ and $v(t) < 0$ when $t > 4$. The total distance $= \int_0^5 |v(t)|\, dt = \int_0^4 12t - 3t^2\, dt + \int_4^5 3t^2 - 12t\, dt = (6t^2 - t^3)\Big|_0^4 + (t^3 - 6t^2)\Big|_4^5 = 32 + (-25 - (-32)) = \boxed{39}$ m.

**5.** [**10 points total**] Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^2$, $y = 2 - x^2$ and the vertical line $x = 0$.

(a) [**2 points**] Sketch $\mathcal{R}$.

(b) [**8 points**] Compute the volume of the solid of revolution obtained by rotating $\mathcal{R}$ around the line $y = -1$.

**Answer.**

*Step 1.* (see a). Find intersection point(s) of the graphs:
$x^2 = 2 - x^2$
$2x^2 = 2$
$x = \pm 1$
Since we are in the first quadrant, the only solution is $x = 1$.

Also, sketch a typical washer here. It is perpendicular to the $x$-axis. The variable is **x**.

*Step 2.* $A(x) = \pi R_x^2 - \pi r_x^2 = \pi(2 - x^2 + 1)^2 - \pi(x^2 + 1)^2 = \pi(3 - x^2)^2 - \pi(x^2 + 1)^2 = \pi(8 - 8x^2)$.

*Steps 3,4.* $V = \pi \int_0^1 (8 - 8x^2)\, dx = \pi(8x - \frac{8}{3}x^3)\Big|_0^1 = \boxed{\frac{16\pi}{3}}$.