MATH 124 Calculus and Analytic geometry I Fall 2005
Practice Midterm I **Name:** ____________________

Instructions: do problems

1a-c; 2, 3a-d, 4a, 5, 7a

in class for this week's worksheet. If you have time, start the last problem. Do not turn in the worksheet! Finish the rest of the practice problems at home. You will have another chance to ask questions about the practice midterm during the TA session on Thursday, Oct. 20.

Note: you must use limits to find derivatives and slopes of tangent lines. No credit will be given for using shortcuts you might have learned outside of the classroom (not to worry: you'll have plenty of opportunity to use these shortcuts later in the course).

Note: the actual midterm is "no notes, books or graphing calculators". You may use your "simple" scientific calculator on the exam. You have to show ALL YOUR WORK on the exam to get credit.

1. Evaluate the following limits.

   (a) $\lim\limits_{x\to 1} \frac{x^2-3x+2}{x-1}$

   Answer: -1.

   (b) $\lim\limits_{x\to 0} \frac{\sqrt{x+1}-\sqrt{1-x}}{x}$

   Answer: 1.

   (c) $\lim\limits_{x\to 0} \frac{\tan x}{x}$

   Answer: 1.

   (d) $\lim\limits_{t\to 3} \frac{t^3-9t}{t^2-9}$

   Answer: 3.

   (e) $\lim\limits_{\theta\to 0} \frac{\sin^2\theta}{\theta}$

   Answer: 0

   (f) $\lim\limits_{h\to 2} \frac{\frac{1}{h}-\frac{1}{h-2}}{h-2}$

   Answer: $-\infty$.

   (g) $\lim\limits_{x\to 2} \frac{x^2-4}{3x^2-2x-8}$

   Answer: $\frac{2}{5}$

   (h) $\lim\limits_{x\to\infty} \frac{\sqrt{17x^4-1}}{x^2+x}$

   Answer: $\sqrt{17}$.

   (i) $\lim\limits_{x\to-\infty} \frac{\sqrt{17x^4-1}}{x^2+x}$

   Answer: $\sqrt{17}$.

   (j) $\lim\limits_{x\to 0} x^3\cos\frac{1}{x}$

   Answer: 0. Hint: Use Squeeze theorem.

2. Evaluate the following limit:

$$\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$$

What is the function for which this limit will compute the slope of the tangent line at the point $x = 2$?

Answer: Lim = 4. The function is $f(x) = x^2$.

3. **(a)** Graph the function $2\ln(t+1) - 1$, compute the domain and range. Label carefully intermediate steps.

Answer: Domain: $t > -1$. Range: $\mathbf{R}$.

(b) What are the vertical asymptotes?

Answer: $t = -1$

(c) Where is this function continuous?

Answer: $t > -1$

(d) What is the rate of change of $2\ln(t+1) - 1$ at the point $t = 0$. Use limits and Problem 6 from Homework assignment 3 to answer this question.

Answer: 2.

(e) Which of the following can be the graph of the derivative of $2\ln(t+1) - 1$?

4. For the following functions, determine horizontal and vertical assymptotes if they exist. For each function state where the function is continuous.

(a) $f(x) = \frac{x^2+1}{x^2-1}$.

Answer: Horizontal: $y = 1$, vertical: $x = 1, x = -1$, function is continuous everywhere except at $x = 1, -1$.

(b) $f(x) = \frac{x}{1+x}$.

Answer: Horizontal: $y = 1$, vertical: $x = -1$, function is discontuous at $x = -1$.

(c) $f(x) = \sqrt{x+1} - \sqrt{x}$.

Answer: Horizontal: $y = 0$, no vertical asymptotes. Function is defined and continuous when $x \geq 0$.

5. Let $P(t)$ denote the population of a community of squirrels at time t in years after the year 1999. Suppose that

$$P(t) = \frac{4000}{1 + Ca^t},$$

where C and a are positive constants and $0 < a < 1$. Suppose also that the population of squirrels in 1999 ($t = 0$) was 1000, and the population of squirrels now ($t = 5$) is 2000.

(a) Determine the constants C and a.

Answer: $C = 3$, $a = 3^{-\frac{1}{5}}$.

(b) When will the population of squirrels be 2500?

Answer. $t = 5\log_3(5) \simeq 7$ years and 4 months.

(c) What is $\lim_{t\to\infty} P(t)$?

Answer. 4000.

6. The price of a stock is modeled by a function $v(t) = Ae^{rt}$ of exponential type, where A and r are constants. Here, t is time in months after January 28, 2003. Assume the stock price is \$200 on January 28, 2003 and the price was \$50 on January 28, 2001 (i.e. exactly 24 months ago).

(a) Find a formula for $v(t)$. Give EXACT answers for A and r.

Answer: $A = 200$, $r = \frac{\ln 4}{24}$;
$v(t) = 200e^{\frac{\ln 4}{24}t} = 200 \cdot 4^{\frac{t}{24}}$

(b) What is the average rate of change in the stock price over the past 24 months? Give and EXACT answer and include units.

Answer: 6.25 \$/m

(c) How long does it take the stock price to triple? (Round your answer to the nearest month.)

Answer: About 19 months.

7. **(a)** Find the equation of the tangent line to the curve $y = x^3 - x$ at the point where $x = r$. For which values of r is this tangent line horizontal?

Answer. Equation (point-slope form): $y - (r^3 - r) = (3r^2 - 1)x - r$. The tangent line is horizontal when the slope is 0. Thus, we have to solve $3r^2 - 1 = 0$. We get $r = 1/\sqrt{3}$, and $r = -1/\sqrt{3}$.

(b) Find an equation of the tangent line to $y = e^x \sin x$ at the point $(0, 0)$.

Answer. $y = x$.

8. Compute and graph the derivative of the function $y = \frac{x}{1+x}$.

Answer: $y'(x) = \frac{1}{(1+x)^2}$. To graph, first do $1/x^2$, then shift to the left by 1.

9. The height $y(t)$ (in feet at time t seconds) of a ball thrown vertically upwards is given by $y(t) = -16t^2 + 128t + 25$. Find the velocity of the ball at time $t = 1$. Find the velocity of the ball when it hits the ground.

Note. Since $y(0) = 25$ the ball is NOT thrown from the ground but from the height of 25 feet.

Answer. When $t = 1$, $y'(t) = 96 feet/s$. When the ball hits the ground, $t = 4 + \frac{\sqrt{281}}{4} \simeq 8.19$, the velocity is $-32(4 + \frac{\sqrt{281}}{4}) + 128 \simeq -134$.